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SMART CITIES Conference

Temporal Networks with Zero Latency

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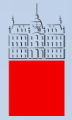
Outline

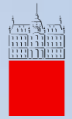
- Temporal Networks
- Temporal Quantities
- Operations on Temporal Quantities
- Violence Network and Terror News Network
- Degrees, Closure and Connectivity

Temporal Networks



- A temporal network is an ordinary network with an attached set of time ponts .
  - The nodes and the links are not necessarily present or active at all times. Also the properties of the links and the nodes can change in time.
  - We denote with ( ) and ( ) the activity sets of time points for the node and for the link , respectively.
- Consistency condition (( ))
  - If a link ( ) is active at the time , also the end-nodes and should be active at that time.

Temporal Quantities



- A notion of a temporal quantity

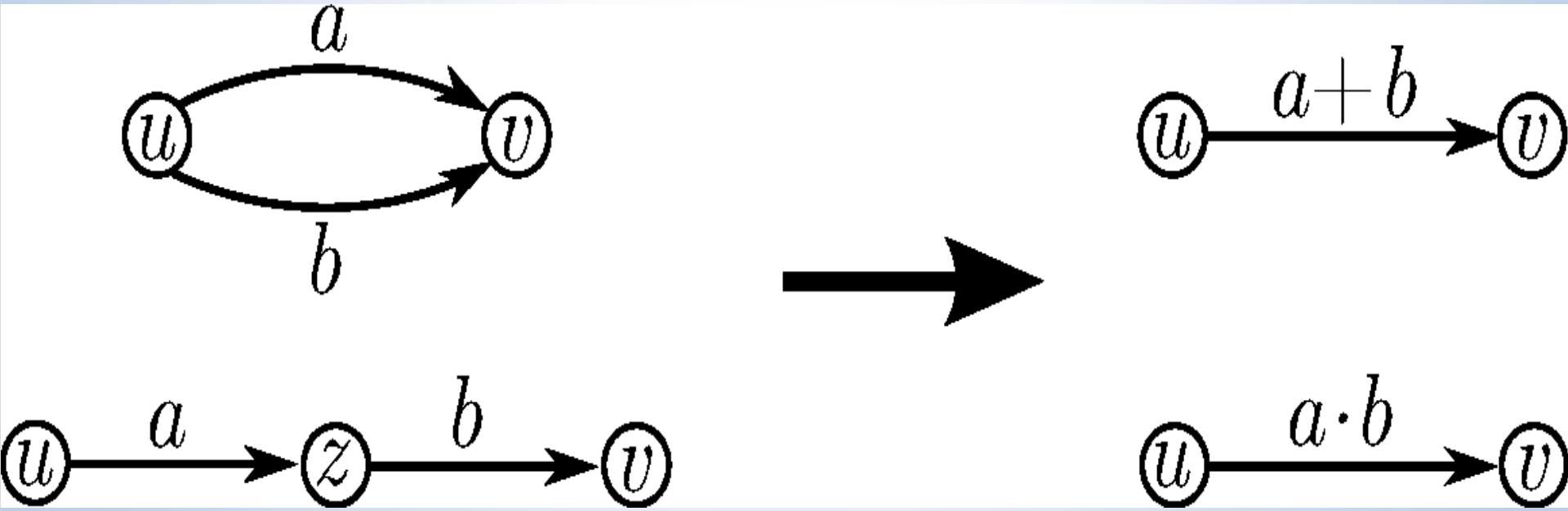
where is the activity set of and is the value of at a time

  - We assume that the values of temporal quantities belong to a semiring .
  - The semiring operations can be extended to the set .

Semiring operations

Combinatorial semiring

- The default semiring is . Other semirings are used in some aplications (reachability, geodesics, Pathfinder, etc.).
- The addition describes the composition of values on parallel paths.
- The multiplication describes the composition of values on sequential paths.



Operations with Temporal Quantities

- The sum of temporal quantities and is defined as

$$( ) \quad \begin{cases} ( ) \\ \\ \\ \\ \end{cases}$$

and .

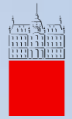
Operations with Temporal Quantities

- The product of temporal quantities and is defined as

( ) ( ) ( )

and .

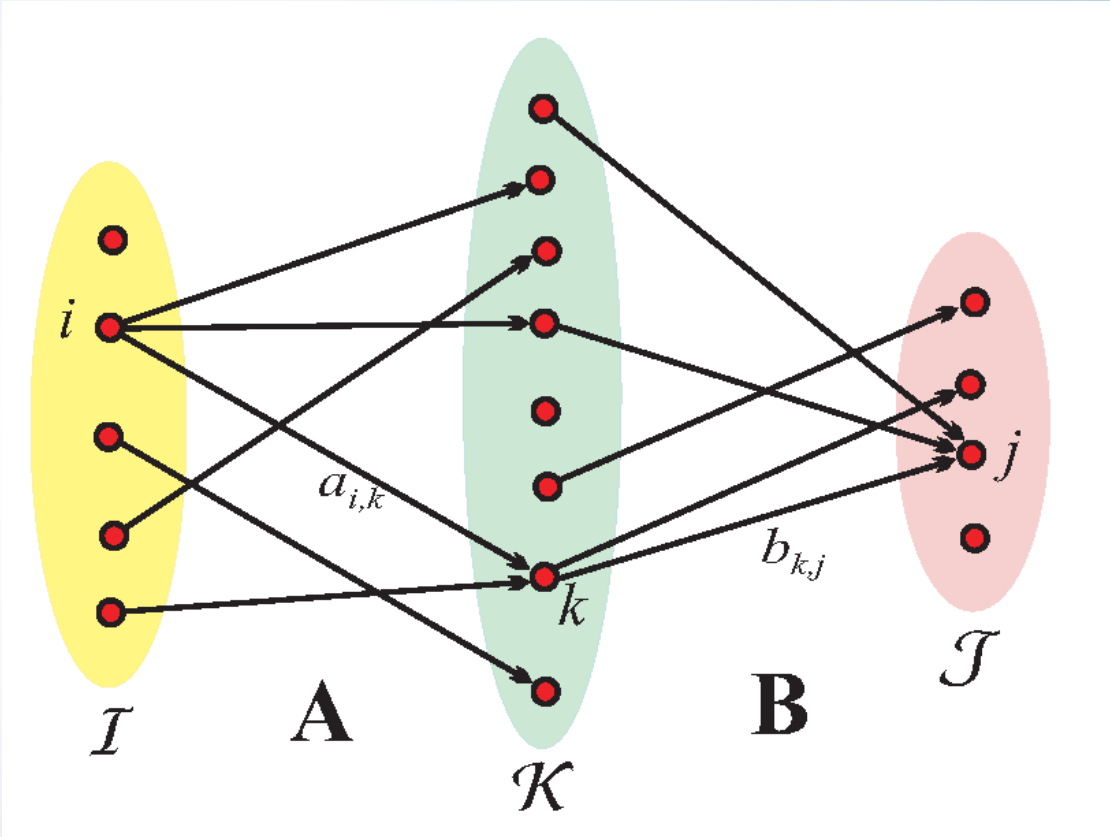
A semiring for temporal quantities



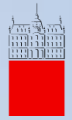
- Define the temporal quantities ( ) and ( ) for all . The structure ( ) is a semiring.
- Also the set of square matrices over this semiring is also a semiring for
  - The addition
  - The multiplication

Matrix multiplication and traveling on networks

For a value to be defined there must exist at least one node such that both a link and a link exist – the transition from the node to the node is possible through the node . Its contribution is .



The Python library TQ

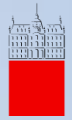


- We limit our approach and describe temporal quantities in terms of a time interval and the value of the quantity in this interval. The value is presumed to be constant on each interval.
- In Python notation:
  - `a = [(1, 5, 2), (6, 8, 1), (11, 12, 3), (14, 16, 2), (17, 18, 5), (19, 20, 1)]`
  - `b = [(2, 3, 4), (4, 7, 3), (9, 10, 2), (13, 15, 5), (16, 21, 1)]`

The temporal quantity has on the interval the value 2, etc.

Outside the specified intervals, it is undefined.

Addition and multiplication



```
a = [(1, 5, 2), (6, 8, 1), (11, 12, 3), (14, 16, 2), (17, 18, 5), (19, 20, 1)]
b = [(2, 3, 4), (4, 7, 3), (9, 10, 2), (13, 15, 5), (16, 21, 1)]
```

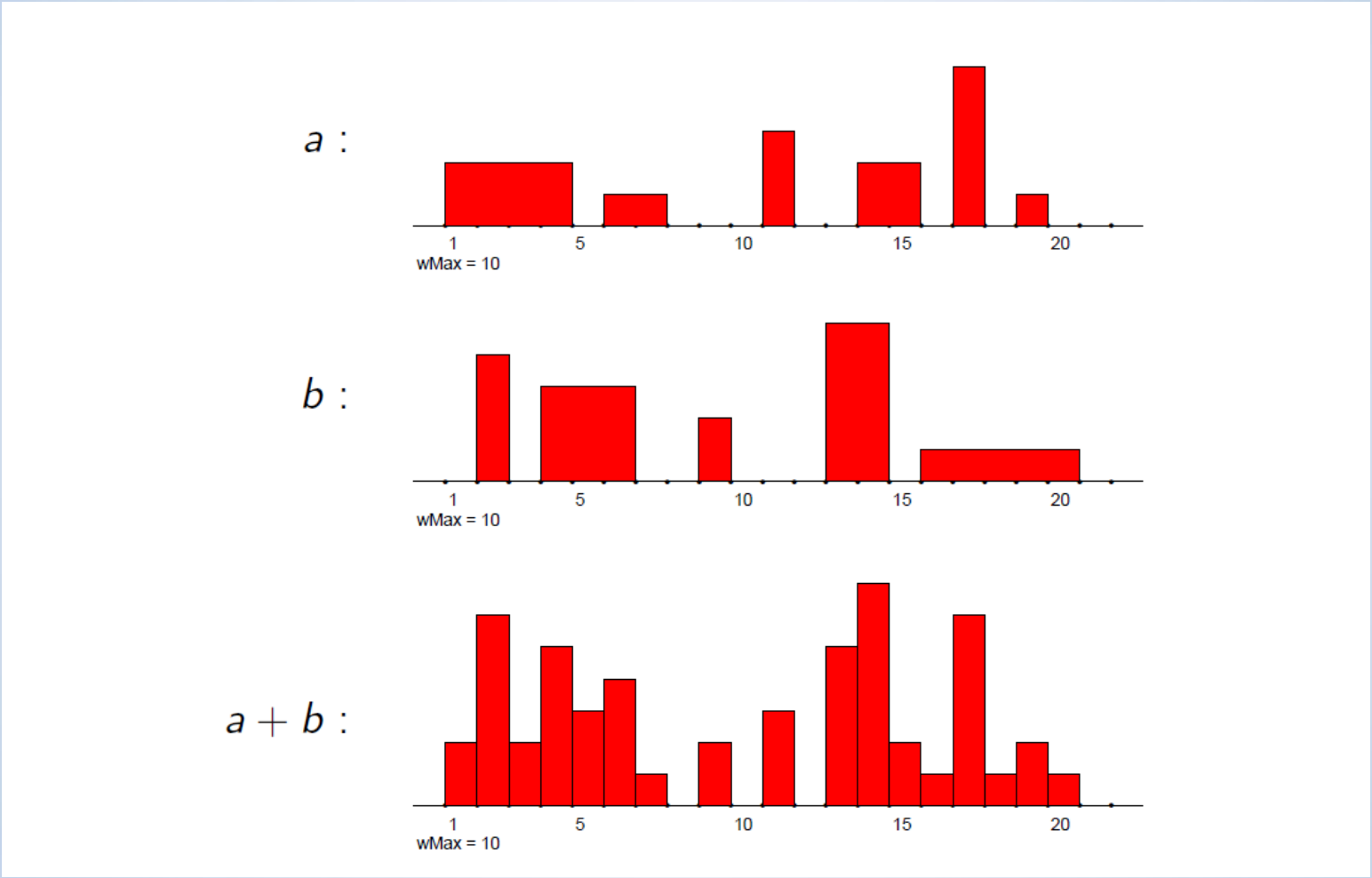
- The sum

```
s = [(1, 2, 2), (2, 3, 6), (3, 4, 2), (4, 5, 5), (5, 6, 3), (6, 7, 4), (7, 8,
1), (9, 10, 2), (11, 12, 3), (13, 14, 5), (14, 15, 7), (15, 16, 2), (16, 17,
1), (17, 18, 6), (18, 19, 1), (19, 20, 2), (20, 21, 1)]
```

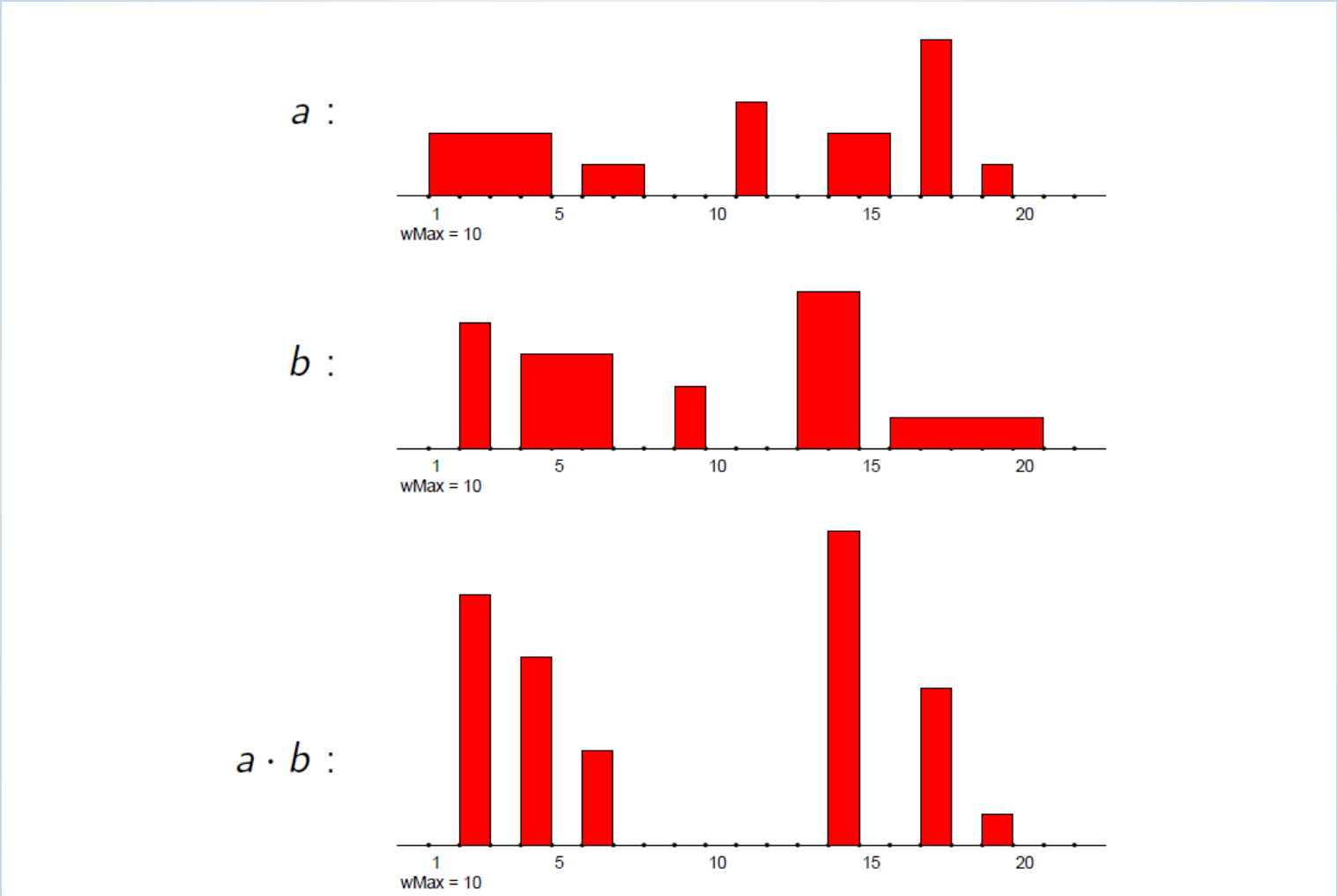
- The product

```
p = [(2, 3, 8), (4, 5, 6), (6, 7, 3), (14, 15, 10),(17, 18, 5), (19, 20, 1)]
```

Addition of temporal quantities



Multiplication of temporal quantities



Activity between groups

- The activity between two groups of nodes and is defined as

( )

  - We illustrate this notion on Franzosi‘s violence temporal network. The data was collected from the journal news in the period from January 1919 to December 1922. It contains the data about violent actions and the counts of interactions between different groups of people.

Activities of police, fascists and all

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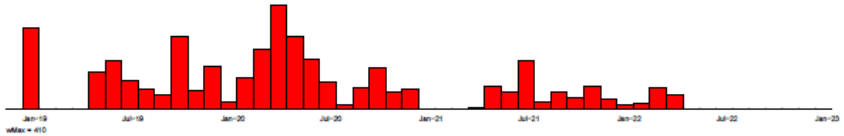
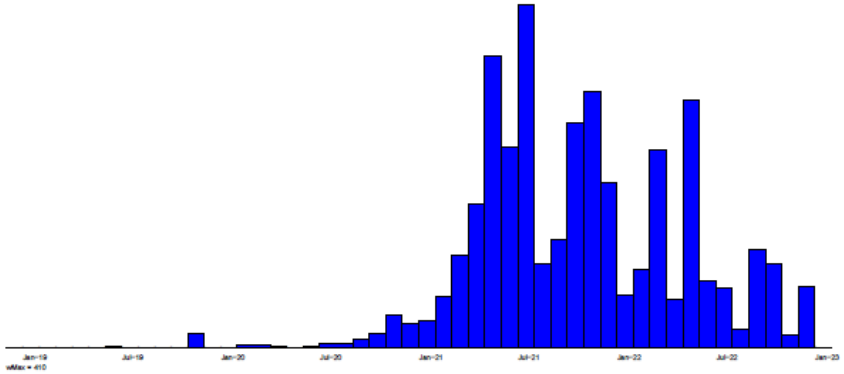
Most of the violent activity in 1919 and 1920 was related to the police.

In the years 1921 and 1922 the violent activities were taken over by the fascists.

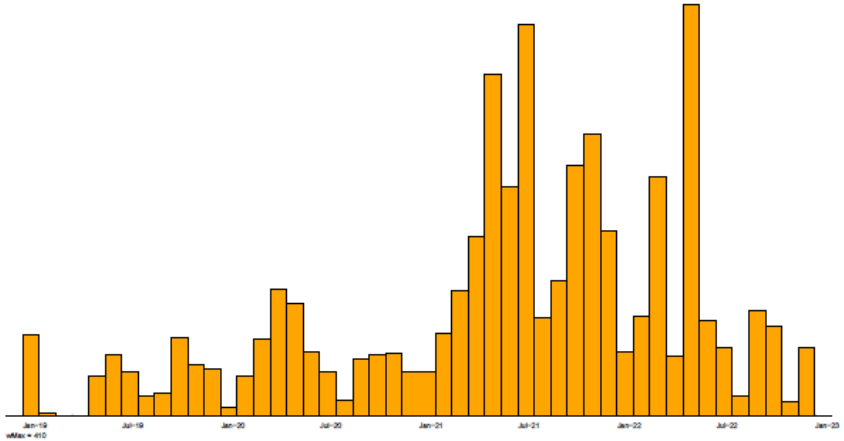
police :



fascists :



all :



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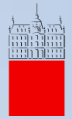
September 11th Reuters terror news



- Reuters terror news network was obtained from the CRA networks produced by Steve Corman and Kevin Dooley at Arizona State University.
- It is based on the news stories about September 11th attack, during the 66 consecutive days.
- The nodes are words, there is an edge between two words iff they appear in the same sentence. The weight of an edge is its frequency.

Terror news network

The 50 most active nodes and the edges with value greater than 10.

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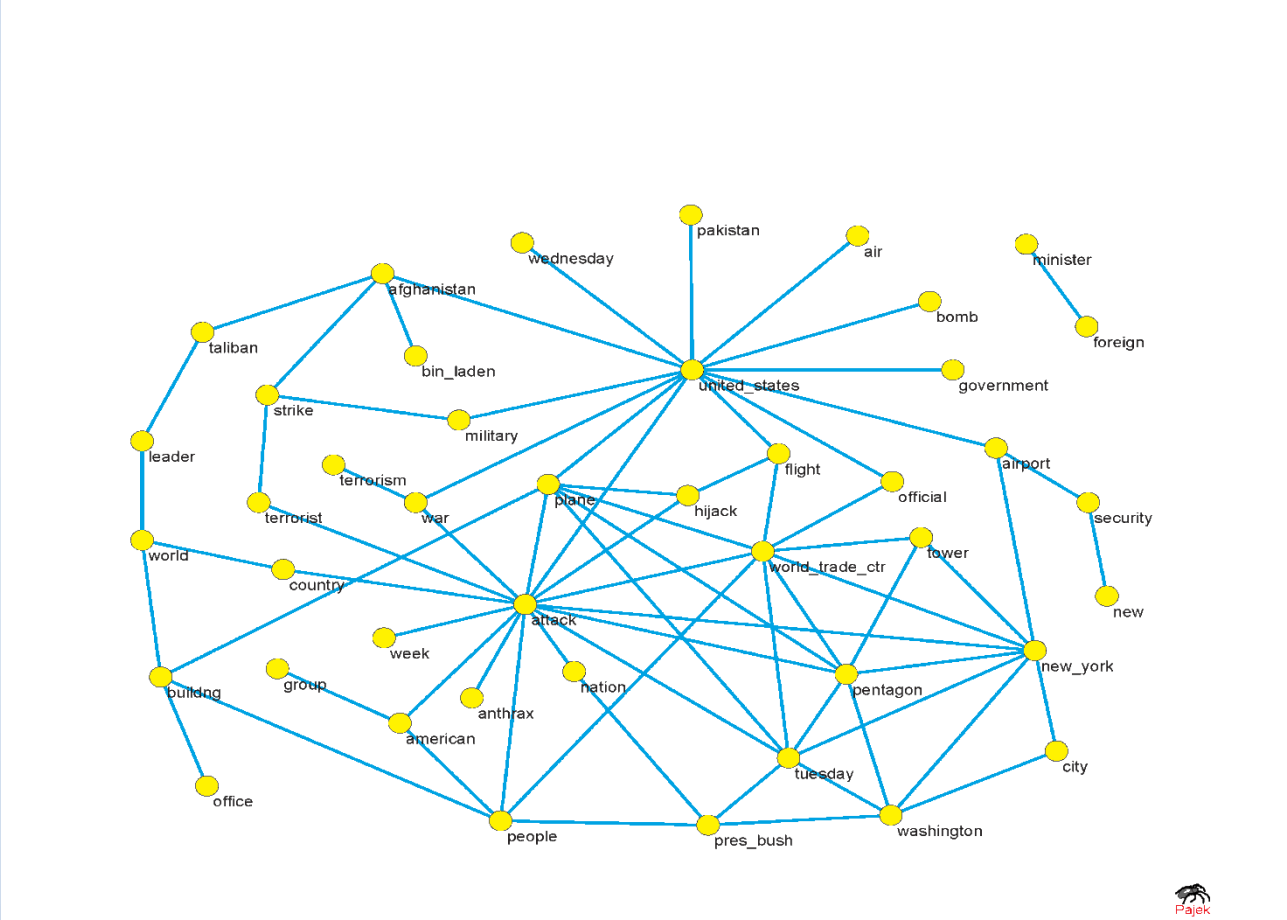

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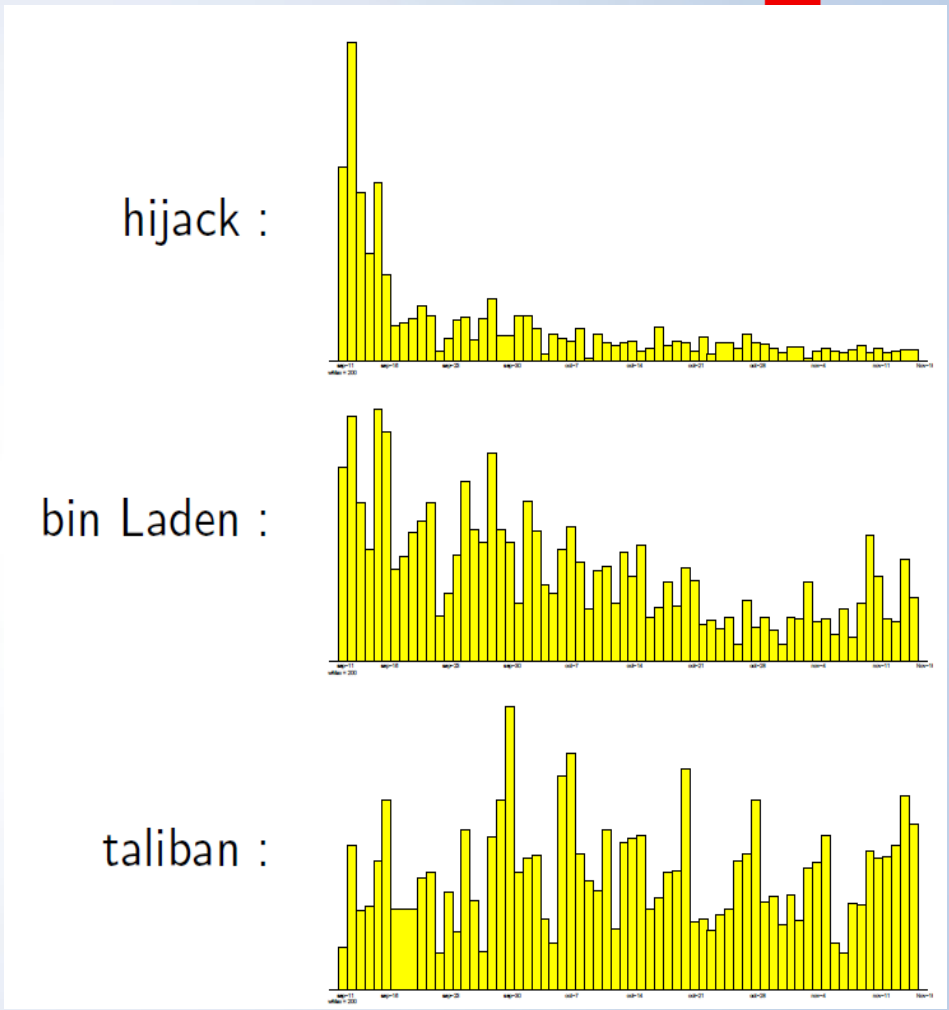
Node activity

- Let be a network matrix.
- Node activity is

( ) { }

- Different types of activity: occasional with several peaks, slowly declining, stationary.



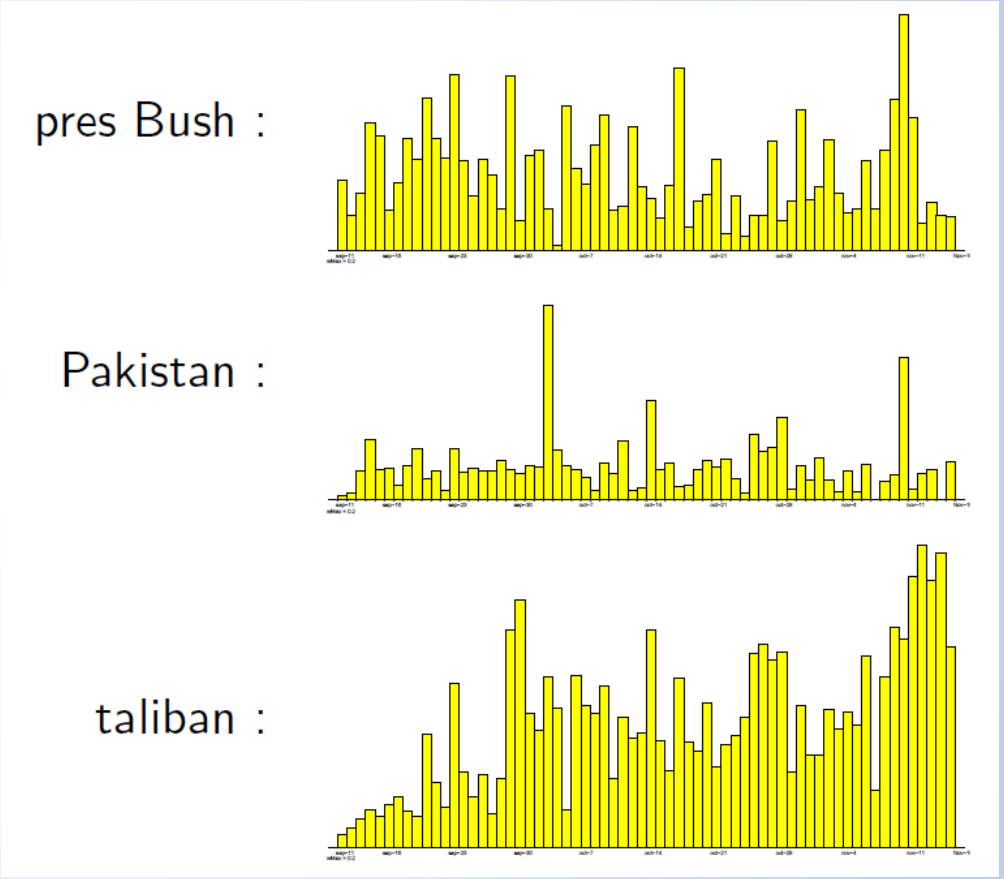
Node attraction

- Node attraction is

  ( ) – ———

- The fraction —— measures the proportion of the activity of that is shared with

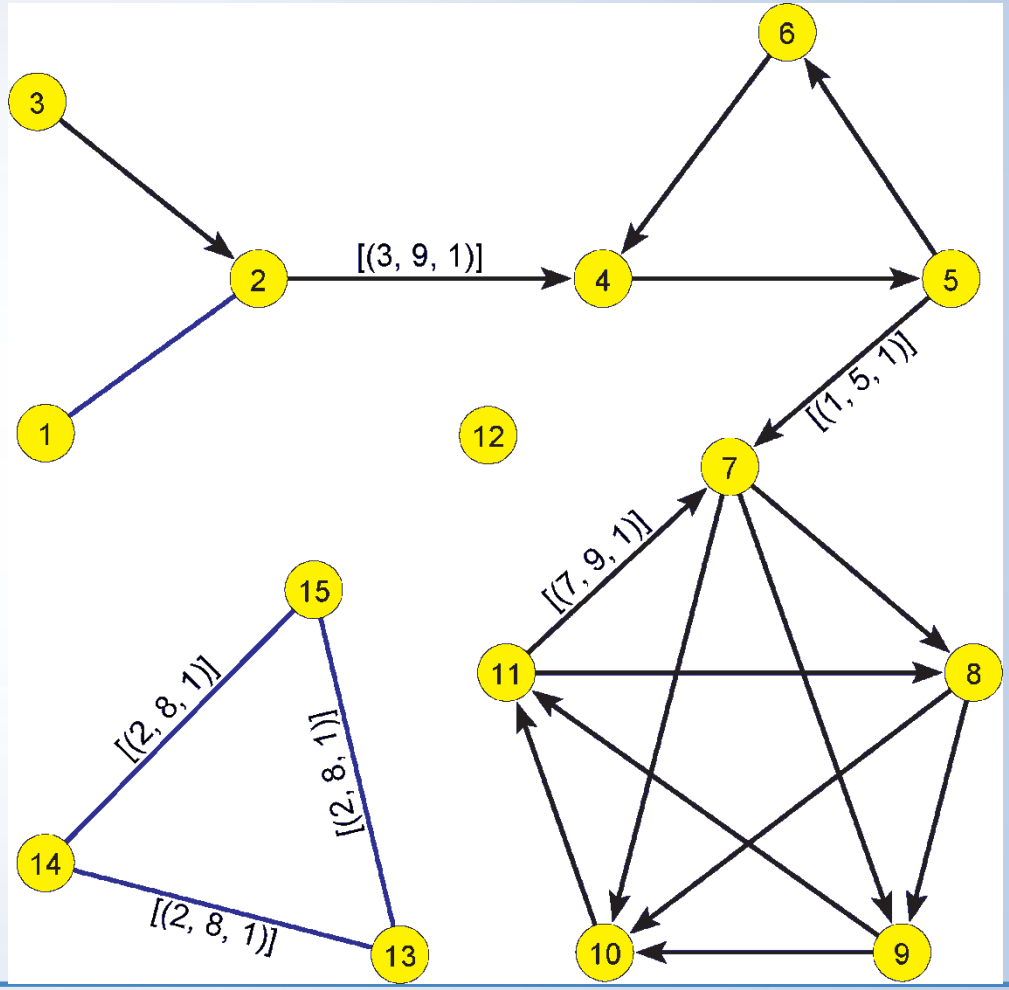
- The attraction of the term „taliban“ is greater at the end of the time window. (Compare to the activity.)



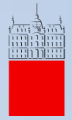
Temporal degrees

- We compute temporal degrees in the same way as for an ordinary graph: and .

- Some indegrees:
  - `4 : [(1, 3, 1), (3, 9, 2)]`
  - `7 : [(1, 5, 1), (7, 9, 1)]`
  - `13 : [(2, 8, 2)]`
- Some outdegrees:
  - `2 : [(1, 3, 1), (3, 9, 2)]`
  - `11 : [(1, 7, 1), (7, 9, 2)]`
  - `13 : [(2, 8, 2)]`



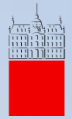
Closures in temporal networks



- When the underlying semiring is closed, a closure operation with the property

  for all , is defined in it.
- For computing the matrix closure, Fletcher‘s algorithm is applied.
- For a temporal network represented with the matrix its transitive closure (in the reachability semiring) determines its reachability relation matrix.

Temporal connectivity

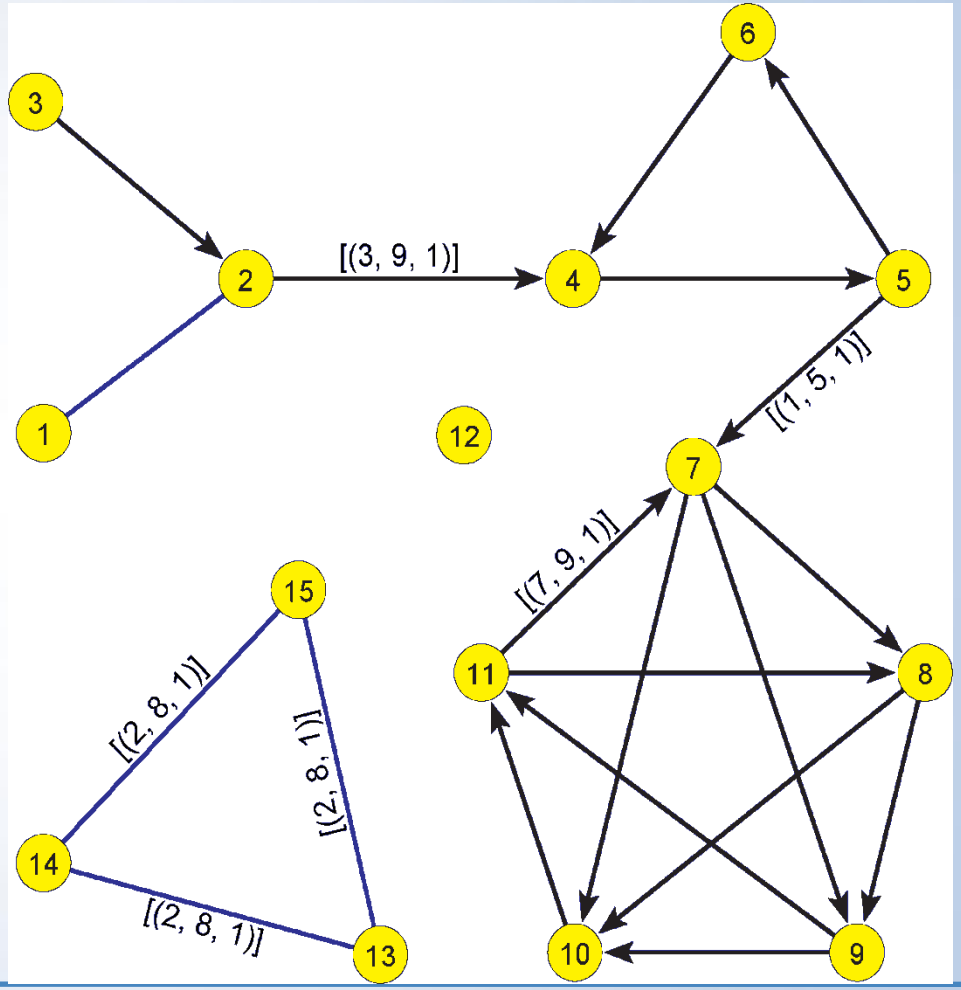


- The weak connectivity temporal matrix is obtained as

- The strong connectivity temporal matrix is

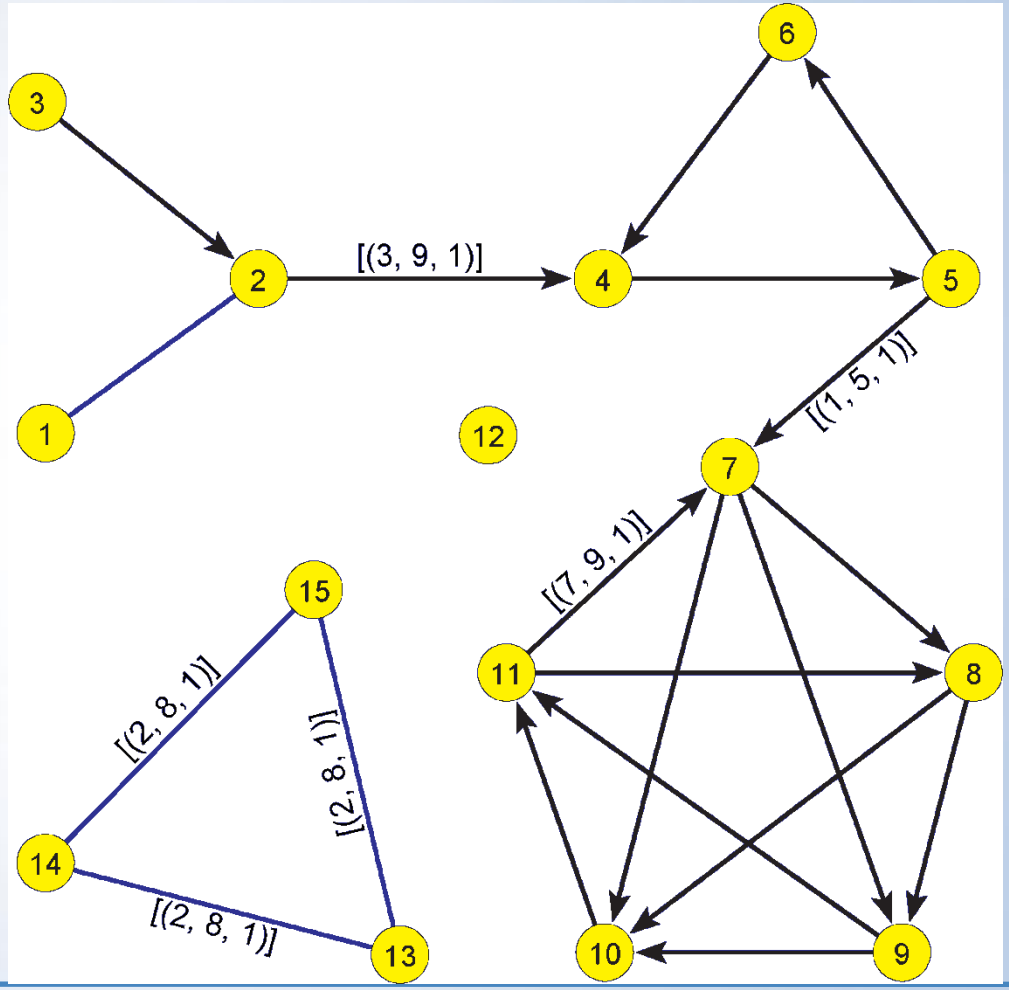
Temporal weak components


```
1:[(1, 3, 1),(3, 5, 2),(5, 9, 3)]
2:[(1, 3, 1),(3, 5, 2),(5, 9, 3)]
3:[(1, 3, 1),(3, 5, 2),(5, 9, 3)]
4:[(1, 3, 4),(3, 5, 2),(5, 9, 3)]
5:[(1, 3, 4),(3, 5, 2),(5, 9, 3)]
6:[(1, 3, 4),(3, 5, 2),(5, 9, 3)]
7:[(1, 3, 4),(3, 5, 2),(5, 9, 5)]
8:[(1, 3, 4),(3, 5, 2),(5, 9, 5)]
9:[(1, 3, 4),(3, 5, 2),(5, 9, 5)]
10:[(1, 3, 4),(3, 5, 2),(5, 9, 5)]
11:[(1, 3, 4),(3, 5, 2),(5, 9, 5)]
12:[]
13:[(2, 8, 6)]
14:[(2, 8, 6)]
15:[(2, 8, 6)]
```



Temporal strong components

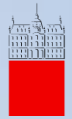

```
1  :  [(1,  9,  1)]
2  :  [(1,  9,  1)]
3  :  []
4  :  [(1,  9,  2)]
5  :  [(1,  9,  2)]
6  :  [(1,  9,  2)]
7  :  [(7,  9,  3)]
8  :  [(1,  7,  4),  (7,  9,  3)]
9  :  [(1,  7,  4),  (7,  9,  3)]
10  :  [(1,  7,  4),  (7,  9,  3)]
11  :  [(1,  7,  4),  (7,  9,  3)]
12  :  []
13  :  [(2,  8,  5)]
14  :  [(2,  8,  5)]
15  :  [(2,  8,  5)]
```



Other results

- Clustering coefficients
- Closeness and betweenness
- Pathfinder
- Spectral centrality measures (Eigenvalues, Bonacich, Katz, Kleinberg, PageRank)

Conclusions



- Alternative to the traditional approach with time slices.
- Program Ianus and the library TQ one of the first programs for the analysis of dynamic networks. Many traditional concepts need to be adapted and new ones developed.
- Extensions to networks with non-zero latency?

Thank You

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