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Collected by Marcin Kamiński and Martin Milanič

Maximum clique for disks of two sizes

by Sergio Cabello

We do not know how hard is finding a largest clique in the intersection graph of disks. Here I would like to propose a more modest, hopefully easier, problem.

Let G be the intersection graph of a family D of disks in the plane. Assume that D is given and contains disks of two different sizes. I am aware of two different algorithms to get a 2-approximation to the maximum clique problem in G. The first algorithm is to find the largest clique for each of the disk sizes independently using the algorithm by Clark, Colbourn and Johnson [14], and return the best. The second algorithm is to use the generic 2-approximation algorithm for arbitrary disks by Ambühl and Wagner [1]. Can we get a 1.99-approximation algorithm?

Upper bound on clique-width

by Konrad Dabrowski

The *clique-width* of a graph G, denoted by $\mathrm{cw}(G)$, is the minimum number of labels needed to construct G by using the following four operations: (1) creating a new graph consisting of a single vertex v with label i; (2) taking the disjoint union of two labelled graphs G_1 and G_2; (3) joining each vertex with label i to each vertex with label j ($i \neq j$); (4) renaming label i to j.

Let G be a graph. Let $\mathcal{G}_P$ be the set of prime induced subgraphs of G and let $\mathcal{G}_C$ be the set of 2-connected induced subgraphs of G. The following two lemmas hold:

Lemma 1 **([17])** *Let G be a graph. Then* $\mathrm{cw}(G) = \max\{\mathrm{cw}(H) \mid H \in \mathcal{G}_P\}$.

Lemma 2 **([8, 29])** *Let G be a graph. Then* $\mathrm{cw}(G) \leq \max\{\mathrm{cw}(H) \mid H \in \mathcal{G}_C\} + 2$.

Is there a function f such that the following holds for every graph G:

$$\mathrm{cw}(G) \leq f(\max\{\mathrm{cw}(H) \mid H \in \mathcal{G}_P \cap \mathcal{G}_C\})\ ?$$

Square root
by Petr Golovach

The *square* G^2 of a graph G is the graph with the vertex set $V(G)$ such that two distinct $u, v \in V(G)$ are adjacent in G^2 if and only if u and v are of distance at most 2 in G. A graph H is a *square root* of G if $G = H^2$. There exist graphs with no square root, graphs with a unique square root, as well as graphs with many square roots. In 1994, Motwani and Sudan [32] showed that the SQUARE ROOT problem, which is that of testing whether a graph has a square root, is NP-complete. This fundamental result triggered a lot of research on the computational complexity of recognizing squares of graphs and computing square roots under the presence of additional structural assumptions. We will present two questions:

1. In [15], Cochefert et al. proved that SQUARE ROOT can be solved in polynomial time on graphs of maximum degree at most 6. Is it possible to solve the problem in polynomial time for graphs of maximum degree at most Δ for some fixed $\Delta \geq 7$? Is there a fixed Δ such that SQUARE ROOT is NP-complete for graphs of maximum degree at most Δ?

2. It was observed in [15] that SQUARE ROOT can be solved in time $O^*(3^{m/3})$, where m is the number of edges of the input graph. Is it possible to solve the problem in time $O^*(c^n)$, where n is the number of vertices?

Cyclability
by Petr Golovach

For a positive integer k, a graph G is *k-cyclable* if every k vertices of G lie on a common cycle; we assume that any graph is 1-cyclable. The *cyclability* of a graph G is the maximum integer k for which G is k-cyclable. Respectively, the CYCLABILITY problem (see [22]) asks for a graph G and a positive integer k, whether G is k-cyclable. Clearly, a graph G is Hamiltonian if and only if its cyclability equals $|V(G)|$. Hence, CYCLABILITY is NP-hard. There is no proof (or evidence) that CYCLABILITY is in NP. The definition of the problem classifies it directly in Π_2^{P}. Is CYCLABILITY Π_2^{P}-complete?

The Dudeney Algorithm
by Andreas M. Hinz

Hanoi graphs H_p^n are the mathematical model for the Tower of Hanoi game with $p \geq 3$ pegs and $n \geq 0$ discs; see [25, Section 5.5] for formal definitions. Whereas topological properties for this and the related class of Sierpiński graphs

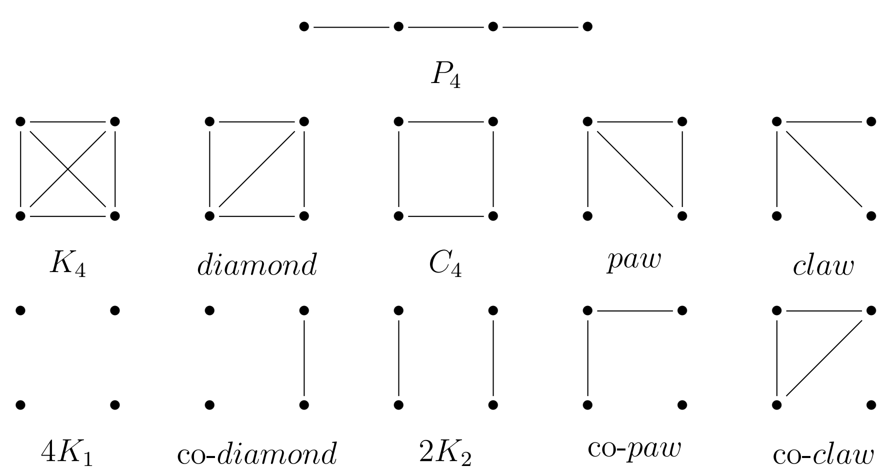
S_p^n [25, Section 4.2] are relatively well known, and the metric of S_p^n is completely understood (cf. [24]), metric properties of Hanoi graphs are extremely difficult to come by. The most prominent open problem is the Frame-Stewart conjecture about the minimality of a certain algorithm for the distance between perfect states, i.e. vertices 0^n and 1^n ([25, Sections 5.1 to 5.4]). Although possibly solved for the case $p = 4$, the so-called "The Reve's puzzle" (cf. [10]), the conjecture is open for larger p.

To understand the structure of Hanoi graphs better, which is complicated by the great choice of "largest disc moves" (see [5]), it would be nice to have precise bounds for the radius and diameter. For instance, the best theoretical result for the latter is $\operatorname{diam}(H_p^n) \leq 2^n - 1$ [25, Proposition 5.36]. Although this bound is tight for Sierpiński graphs $\operatorname{diam}(S_p^n) = 2^n - 1$ [25, Corollary 4.9], numerical experiments show that it is extremely off the mark for Hanoi graphs, where, e.g., we have $\operatorname{diam}(H_4^{15}) = 130$, which is, of course, far away from $2^{15} - 1 = 32767$.

Coloring graphs without 4-vertex induced subgraphs
by Chính T. Hoàng

Let L be a set of graphs. *Free*(L) is the set of graphs that do not contain any graph in L as an induced subgraph. It is known that if L is a set of four-vertex graphs, then the complexity of the coloring problem for *Free*(L) is known with three exceptions: $L = \{\text{claw}, 4K_1\}$, $L = \{\text{claw}, 4K_1, \text{co-diamond}\}$, and $L = \{C_4, 4K_1\}$. For each of the above three problems, determine whether it is NP-complete or polytime solvable.

A partial solution to the first problem is given in [21]: the authors give a polytime algorithm to color a $4K_1$-free line graph.



Fast exponential algorithm for Maximum Acyclic Subgraph
by Eunjung Kim

Given a directed graph $D = (V, A)$ on n vertices, the MAXIMUM ACYCLIC SUBGRAPH problem, also called FEEDBACK ARC SET, is to find an acyclic subdigraph of G containing as many arcs as possible. This problem is equivalent to the ORDERING 2-CSP. An instance of ORDERING 2-CSP consists of n variables X and a collection ϕ of constraints of arity two, i.e. a tuple of two variables of X. A constraint $C = (x_i, x_j)$ with $x_i, x_j \in X$ is satisfied by an ordering $\sigma : X \to [n]$ if $\sigma(x_i) < \sigma(x_j)$. The goal of ORDERING 2-CSP is to find an ordering σ of X satisfying as many constraints of ϕ as possible. This problem can be naturally extended to ORDERING k-CSP for $k \geq 3$: each constraint is a tuple of (at most) k variables and a constraint C is satisfied by σ if the ordering of variables in the tuple C is respected by σ.

It is fairly straightforward to obtain a $O^*(2^n)$-time algorithm for ORDERING k-CSP with $k = 2$ or 3 using dynamic programming, e.g., see [7]. In contrast, it is proved in [27] that there is no $2^{o(n \log n)}$-time algorithm for $k \geq 4$ unless `Exponential Time Hypothesis` [26] fails. This means that a significantly better algorithm than the naive enumeration of all possible orderings for ORDERING 4-CSP would imply $2^{o(n)}$-time algorithm for 3CNF-SAT, which would be a huge breakthrough and widely believed to be unlikely. On the other hand, it is not known whether there is c^n-time algorithm for $c < 2$ even for the case $k = 2$, which is equivalent to MAXIMUM ACYCLIC SUBGRAPH.

Therefore, we pose it as an open question: Is there an algorithm to solve MAXIMUM ACYCLIC SUBGRAPH, equivalently ORDERING 2-CSP, in time $O(c^n)$ with $c < 2$? Or, is this unlikely under certain complexity assumption? The same question for ORDERING 3-CSP is also interesting.

Equipartitions of graphs
by Jurij Kovič

This problem is about replacing a minimal number of edges in a given graph with the same number of other edges so that as a result we get a disjoint union of isomorphic graphs. More precisely:

Let G be a graph with $m = |V(G)|$ vertices and $n = |E(G)|$ edges and let $gcd(m, n) > 1$. For any integer $k > 1$ dividing $gcd(m, n)$ let $e_k(G)$ be the smallest number of edges $e(i, j) \in E(G)$ we have to delete and replace with the same number of edges of the complementary graph $G^C = K_m - G$ to get a graph G_k that can be expressed as a disjoint union of k isomorphic graphs. Any such graph G_k is called an *equipartition* of G into k parts and $e_k(G)$ is called the *equipartion number* of the graph G. Now the following questions may be asked:

1. Choose a divisor $k > 1$ of $gcd(m, n) > 1$. Find an algorithm which takes as the Input a given graph G with $m = |V(G)|$ vertices and $n = |E(G)|$ edges, and gives as the Output an equipartition G_k of G into k isomorphic graphs. We

also demand that this algorithm replaces only $e_k(G)$ edges of G with the same number of edges of the complementary graph $G^C = K_m - G$.

2. Choose m, n, k such that $gcd(m, n) > 1$ and $k > 1$ divides $gcd(m, n)$. Find the tight upper bounds for the equipartition numbers $e_k(G)$ for all the graphs G with $m = |V(G)|$ vertices and $n = |E(G)|$ edges.

If the general problems 1. and 2. turn out to be too hard, maybe at least the simplest special case for $k = 2$ may be studied. It can be formulated as follows: Let G be a graph with an even number m of vertices and an even number n of edges. Find an algorithm that replaces the smallest possible number $e_2(G)$ of edges of G with the same number of edges of the complement $G_C = K_m - G$ to get a disjoint union of two copies of isomorphic graphs with $m/2$ vertices and $n/2$ edges.

You may also add some additional conditions for the graph G or for the graphs of the equipartition. For example, you may study a transformation of the cycle $G = C_{ka}$ into a disjoint union of k cycles C_a and try to show that it is enough to replace $2(k-1)$ edges to do this.

I do not know if any such problems have been studied so far. I also do not know of any possible applications. Yet I find the problem of equipartition of graphs both interesting and beautiful.

Well-quasi-order versus Clique-width
by Vadim Lozin

The celebrated result of N. Robertson and P.D. Seymour states that the set of all finite graphs is well-quasi-ordered by the minor relation [35]. However, the induced subgraph relation is not a well-quasi-order. In particular, the set of all chordless cycles form an infinite antichain with respect to this relation. On the other hand, it may become a well-quasi-order when restricted to graphs in special classes, which is the case, for instance, for P_4-free graphs [19]. It is interesting to observe that the clique-width of P_4-free graphs is bounded by 2.

J. Daligault, M. Rao and S. Thomassé ask in [18] if the clique-width is bounded in every hereditary class of graphs which is well-quasi-ordered by the induced subgraph relation. Recently, together with I. Razgon and V. Zamaraev, we showed that this is not generally true [30]. However, we believe and conjecture that the answer is positive for hereditary classes defined by finitely many forbidden induced subgraphs.

If a hereditary class X defined by finitely many forbidden induced subgraphs is well-quasi-ordered by the induced subgraph relation, then the clique-width of graphs in X is bounded by a constant.

Some results related to this conjecture can be found in [3, 4].

The readability of bipartite graphs
by Paul Medvedev

Let $G = (V, E)$ be a bipartite graph with a given bipartition of its vertex set $V(G) = V_s \cup V_p$. (We will also use the notation $G = (V_s, V_p, E)$.) A *labeling* of G is a function ℓ assigning a string to each vertex such that all strings have the same length, denoted by $len(\ell)$. Given two strings x and y, we say that x *overlaps* y if there is a nonempty suffix of x that is equal to a nonempty prefix of y. An *overlap labeling of* G is a labeling ℓ of G such that for all $u \in V_s$ and $v \in V_p$, $(u, v) \in E$ if and only if the strings $\ell(u)$ and $\ell(v)$ overlap (that is, if some nonempty suffix of $\ell(u)$ equals some nonempty prefix of $\ell(v)$).

The *readability* of G, denoted by $r(G)$, is the smallest nonnegative integer r such that there exists an overlap labeling of G of length r. It follows from results of Braga and Meidanis [11] that every bipartite graph G has $r(G) \leq 2^{\Delta(G)} - 1$, where $\Delta(G)$ denotes the maximum degree of a vertex in G (see [13]). Here we pose an open problem from [13]: Determine the computational complexity status of computing the readability of a given bipartite graph $G = (V_s, V_p, E)$.

Note that there is no restriction on alphabet size. The cases $r(G) \leq 1$ and $r(G) \leq 2$ are known to be polynomial. For every constant $k \geq 3$, the complexity of the problem of testing if $r(G) \leq k$ is open.

CIS graphs
by Martin Milanič

A clique in a graph is a set of pairwise adjacent vertices, and a stable set is a set of pairwise non-adjacent vertices. A clique (resp., stable set) is said to be maximal if it is not contained in any larger clique (resp., stable set). The maximum size of a clique (resp., stable set) in a graph G is denoted by $\omega(G)$ (resp., $\alpha(G)$), the chromatic number by $\chi(G)$, and the complement of G by $\overline{G}$.

A *CIS* graph is a graph in which every maximal stable set and every maximal clique intersect (CIS stands for "Cliques Intersect Stable sets"). The study of CIS graphs is rooted in observations of Berge [6] and Grillet [23] (see [36]). Here we present two problems:

1. Determine the computational complexity status of recognizing CIS graphs.
2. [Dobson et al. [20]] Does every CIS graph G satisfy $\alpha(G)\omega(G) \geq |V(G)|$?

The problem of recognizing CIS graphs is believed to be co-NP-complete [36], conjectured to be co-NP-complete [37], and conjectured to be polynomial [2]. For further background on CIS graphs, see, e.g., [9]. Partial results are known. Question 2 has an affirmative answer in the case of:

- *vertex-transitive graphs* (a vertex-transitive graph is CIS if and only if both G and $\overline{G}$ are well-covered, and $\alpha(G)\omega(G) = |V(G)|$, see [20]),
- *perfect graphs* (which satisfy the inequality independently of being CIS),

- *triangle-free graphs* (every CIS triangle-free graph is P_4-free, hence perfect),
- *C_4-free graphs* (this follows from a simple observation that every clique in a C_4-free graph that intersects all maximal stable sets is *simplicial*, that is, it equals the closed neighborhood of some vertex).

To answer Question 2 affirmatively, it would be enough to show that for every CIS graph G, either $\chi(G) = \omega(G)$ or $\chi(\overline{G}) = \omega(\overline{G})$ (which also seems to be open).

Quadratic knapsack problem on planar graphs
by Joachim Schauer

Let $G = (V, E)$ be a graph and let p_i and w_i be rational numbers ≥ 0. Moreover the quadratic profits p_{ij} are strictly positive rational numbers whenever $(i, j) \in E$, otherwise they are 0. By the quadratic knapsack problem on G we denote the following variant of the classical QKP:

$$(QKP) \qquad \max \sum_{i=1}^{n} p_i x_i + \sum_{(i,j)\in E} p_{ij} x_i x_j \tag{1}$$

$$\text{s.t.} \sum_{i=1}^{n} w_i x_i \leq c \tag{2}$$

$$x_i \in \{0, 1\}, \quad i = 1, \ldots, n. \tag{3}$$

What is the computational complexity of the problem when G is planar: does there exist and $\mathcal{FPTAS}$ or is it stongly $\mathcal{NP}$-hard?

- [34] proved that the problem is inapproximable whenever p_{ij} and p_i are allowed to be negative and G is a star. They also proved strong $\mathcal{NP}$-hardness when G is vertex series-parallel (a graph class that contains all complete bipartite graphs).
- The problem is related to a long standing open problem: what is the computational complexity of the densest-k subgraph problem on planar graphs ([16]). Clearly this problem is much more restrictive than QKP.
- [33] could show strong $\mathcal{NP}$-hardness for 3-book embeddable graphs. Note that this graph class is related to planar graphs, however it is not a minor closed class of graphs.

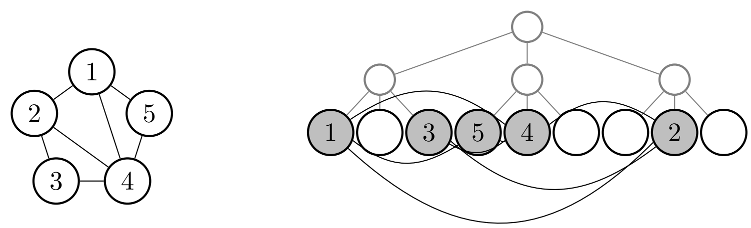
Data arrangement problem on regular trees
by Eranda Çela, Rostislav Staněk, Joachim Schauer

Let $G = (V, E)$, where $|V(G)| = n$ be a so-called *guest* graph and let $d \geq 2$ be fixed. Furthermore, let the so-called *host* graph be a d-regular tree T of height $h = \lceil \log_d n \rceil$ and let B be the set of its leaves. The *data arrangement problem on regular trees* (*DAPT*) asks for an arrangement ϕ that minimizes the objective value $OV(G, d, \phi)$.

$$OV(G, d, \phi) := \sum_{(u,v) \in E} d_T\big(\phi(u), \phi(v)\big), \tag{4}$$

where $\phi\colon V(G) \to B$ is an injective embedding and $d_T\big(\phi(u), \phi(v)\big)$ denotes the length of the unique $\phi(u)$-$\phi(v)$-path in the d-regular tree T.

Example 1 *Let $G = (V, E)$ be the guest graph depicted in Figure 1a and let $d = 3$. The height of the host graph T is $h = \lceil \log_d n \rceil = \lceil \log_d 5 \rceil = 2$.*



(a) Guest graph. (b) Host graph.

Figure 1: Example 1.

Let ϕ bet the arrangement depicted in Figure 1b. We have to count 2 for the edge $(4, 5)$ and 4 for all remaining edges. Thus $OV(G, 3, \phi) = 26$.

Luczak and Noble [31] have shown that this problem is $\mathcal{NP}$-hard for every fixed $d \geq 2$. Çela, Staněk and Schauer [12] proved that the problem remains NP-hard even if the guest graph $G = (V, E)$ is restricted to be a tree. Is the problem solvable in polynomial time if both the guest and the host graph are binary regular trees?

Polynomial number of cycles
by Jean-Florent Raymond

Is there a polynomial p such that every graph G not containing an induced subdivision of $2 \cdot K_3$ has at most $p(|V(G)|)$ induced cycles?

A familly of graphs with an exponential number of induced cycles is $\{G_n\}_{n \geq 6}$, where for every $n \in \mathbb{N}$, $n \geq 6$, G_n is obtained from C_n by doubling every edge and then subdividing every edge. However each of these graphs contains an induced subdivision of $2 \cdot K_3$.

For every $n \in \mathbb{N}$, let H_n be the graph obtained from 5 (disjoint) independent sets $I_1, \ldots I_5$ each of order n by adding all possible edges between I_j and I_{j+1} for every $j \in \{1, 2, 3, 4\}$, and between I_5 and I_1. Every graph of the family $\{H_n\}_{n \in \mathbb{N}}$ does not contain an induced subdivision of $2 \cdot K_3$. In this familly the number of induced cycles of each graph is upper bounded by a polynomial function $p = O(n^5)$ of its number of vertices.

There is a positive answer to the question in any class of graphs where the maximum degree is upper bounded by a polylogarithmic function of the order. The answer is also positive if we replace "not containing an induced subdivision of $2 \cdot K_3$" by "not containing a subdivision of $2 \cdot K_3$" in the statement of the question.

Semitransitive orientations of dart-transitive graphs
by Steve Wilson

For the description of the problem, please see:
https://conferences.matheo.si/conferenceDisplay.py/getPic?picId=33&confId=6

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