

Izzivi poučevanja finančne matematike v angleščini

Aleš Toman

Konferenca slovenskih matematikov 2025

Koper, 12. september 2025

Opredelitev FM: Učni načrt matematike v gimnazijah

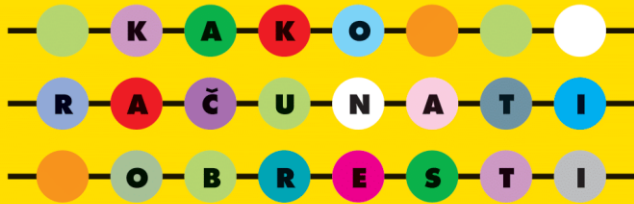


Dijaki/dijakinje:

- razlikujejo **navadno** in **obrestno** obrestovanje,
- razlikujejo med **konformno** in **relativno** obrestno mero,
- uporabijo **načelo ekvivalence glavnice**,
- poiščejo realne primere obrestovanja, napovejo pričakovanja in se odločijo na osnovi simulativnih izračunov,
- izračunajo **anuiteto** in izdelajo **amortizacijski načrt**.

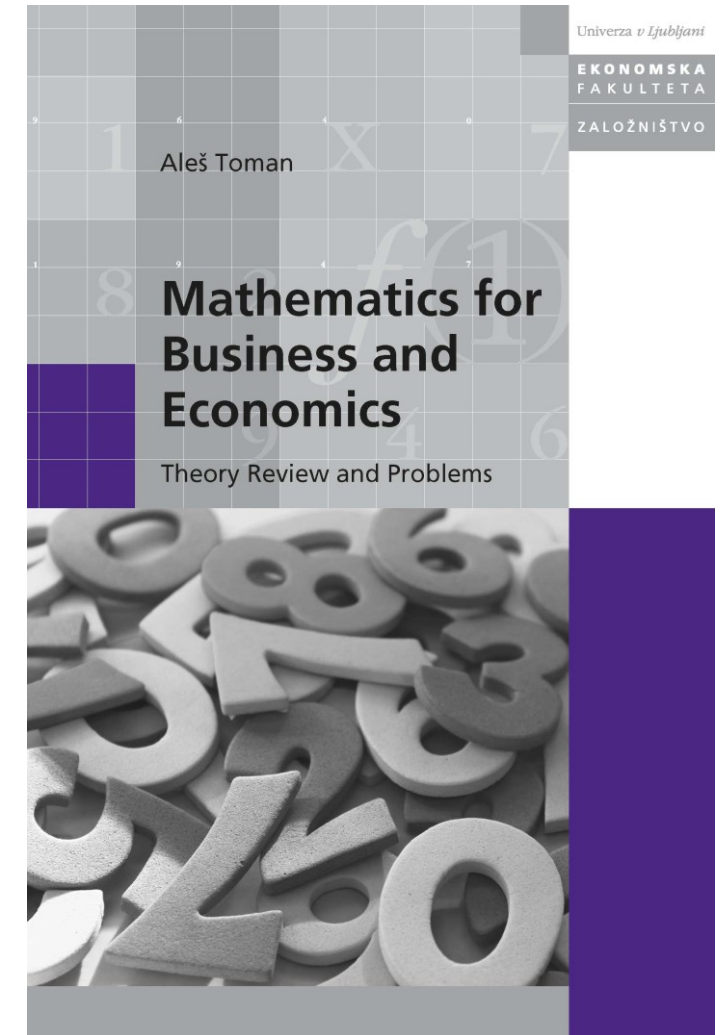
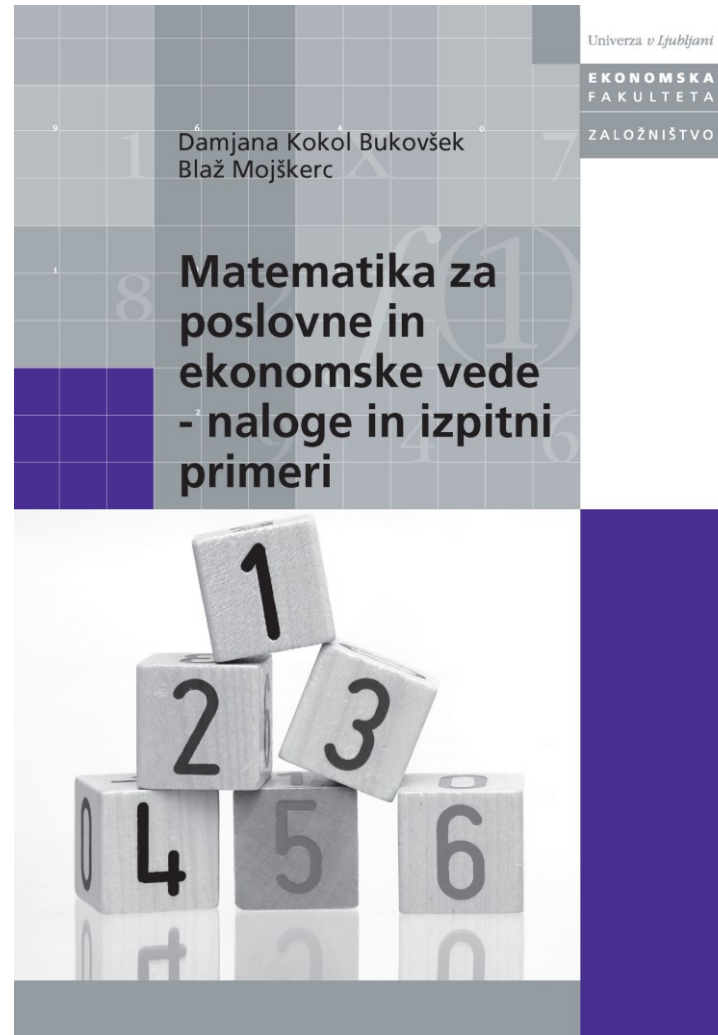
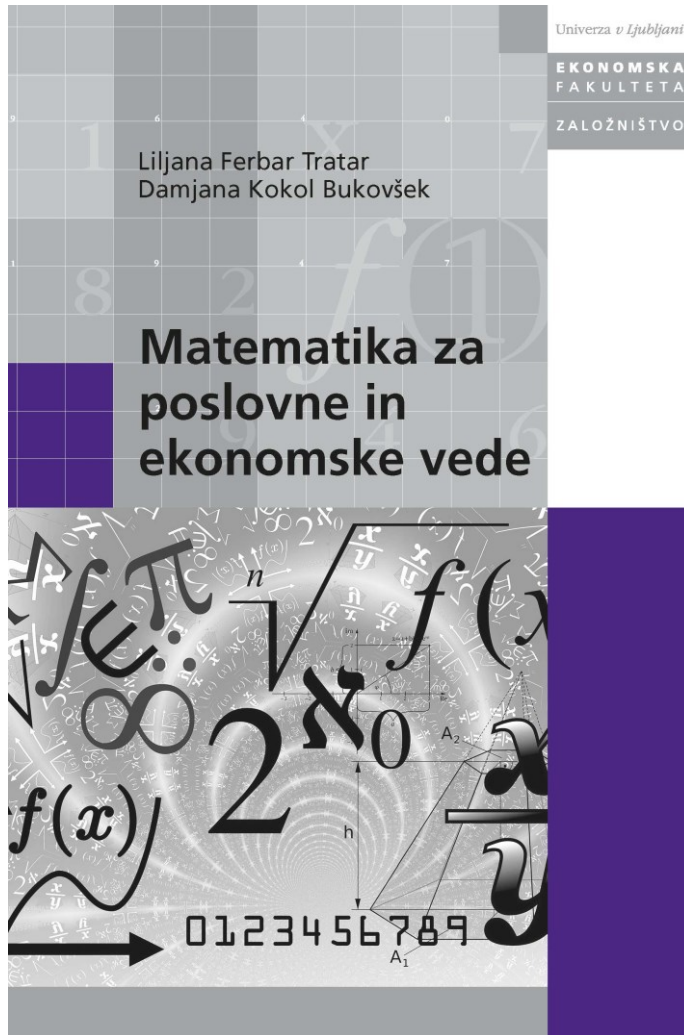
Priporočila Banke Slovenije in drugi viri

Jože Andrej Čibej



- Glavnici G_Z in G_K
- Obrestna mera p %
- Znesek obresti $o = \frac{G \cdot p \cdot m}{1200}$ ali $o = \frac{G \cdot p \cdot d}{36500}$
- Čas obrestovanja $t = \frac{m}{12}$ ali $t = \frac{d}{365}$
- Letni obrestni faktor $r = 1 + \frac{p}{100}$

Učbeniki za matematiko (1. letnik) na Ekonomski fakulteti



Delna rešitev (2019) *When in Rome ...*

Matematika	Mathematics
Glavnica G	Balance B
Obrestna mera p %	Interest rate R (e.g., $R = 5\% = 0.05$)
Obresti o	Interest amount I
Navadno obrestovanje	Simple interest (Simple compounding)
$o = \frac{G \cdot p \cdot m}{1200} = \frac{G \cdot p \cdot d}{36500}$	$I = B \cdot R \cdot T$, where $T = \frac{m}{12} = \frac{d}{365}$ is time in years
Kredit, posojilo	(Instalment) loan
Anuiteta	Instalment amount (Annuity)

Delna rešitev (2019)

When in Rome ...

Matematika	Mathematics
Obrestno obrestovanje	Compound interest (compounding)
Letni obrestni faktor $r = 1 + \frac{p}{100}$	Yearly/Annual interest factor $k = 1 + R$
Relativna obrestna mera	Relative interest rate
$r = 1 + \frac{p}{1200}$ ali $r = 1 + \frac{p}{36500}$	$k = 1 + \frac{R}{12}$ or $k = 1 + \frac{R}{365}$
Konformna obrestna mera	Conform interest rate
$r = \sqrt[12]{1 + \frac{p}{100}}$ ali $r = \sqrt[365]{1 + \frac{p}{100}}$	$k = \sqrt[12]{1 + R}$ or $k = \sqrt[365]{1 + R}$

Mednarodna ekonomska olimpijada (2022–)



INTERNATIONAL
ECONOMICS
OLYMPIAD



A company takes, at the "zero" time instant, a **loan** equal to 71 money units and plans to pay the **first month** 15 money units, the second 28 and the third 30, that is $P_1 = 15$, $P_2 = 28$, $P_3 = 30$

(a) (10 rp) Using the above procedure calculate the IRR for this payment flow. Use **$r_0 = 0.14$, applied annually**, and increase it with step $h = 0.01$ up to the value $r = 0.16$.

(b) (10 rp) If the company, instead of following this payment policy, decides to give 25 money units on the third month and some amount of money the fifth month, what should this amount be, in order the whole loan to be repaid? As an annual interest rate, **use the IRR we found before.**

(c) (10 rp) The company has started to repay the loan by 5 equal monthly payments, covering both principal and interest and with an **interest rate equal to the IRR, found before**. Suddenly after the second payment, the interest rate is increased by 3%. The government, in order to help, decides to cover the 25% of the loan which has not been paid yet and to extend the number of total payments from 5 to 8. Find the amount of each equal monthly payment, the company must pay from now on, to repay the loan. **Relativna ali konformna obrestna mera?**

Solution

(a) (10 points) **Relativna** ali konformna obrestna mera?

For $r_0=0,14$ we have:

$$\begin{aligned} NPV(0,14) &= P_0 \cdot \left(1 + \frac{0,14}{12}\right)^0 + P_1 \cdot \left(1 + \frac{0,14}{12}\right)^{-1} + P_2 \cdot \left(1 + \frac{0,14}{12}\right)^{-2} + P_3 \cdot \left(1 + \frac{0,14}{12}\right)^{-3} \\ &= 0,158975 \end{aligned}$$

For $r_0=0,15$ we have:

$$\begin{aligned} NPV(0,15) &= P_0 \cdot \left(1 + \frac{0,15}{12}\right)^0 + P_1 \cdot \left(1 + \frac{0,15}{12}\right)^{-1} + P_2 \cdot \left(1 + \frac{0,15}{12}\right)^{-2} + P_3 \cdot \left(1 + \frac{0,15}{12}\right)^{-3} \\ &= 0,0302743 \end{aligned}$$

For $r_0=0,16$ we have:

$$\begin{aligned} NPV(0,16) &= P_0 \cdot \left(1 + \frac{0,16}{12}\right)^0 + P_1 \cdot \left(1 + \frac{0,16}{12}\right)^{-1} + P_2 \cdot \left(1 + \frac{0,16}{12}\right)^{-2} + P_3 \cdot \left(1 + \frac{0,16}{12}\right)^{-3} \\ &= -0,09806 \end{aligned}$$

Annual Percentage Rate



APR

Generally measures
**interest charged when
you borrow money**

Usually associated with
credit accounts

vs.

Annual Percentage Yield
Annual Effective Rate



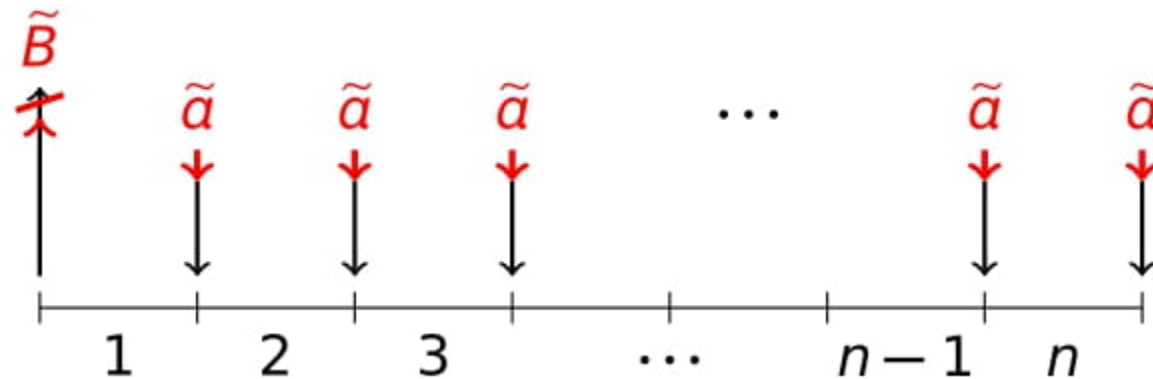
APY

Generally measures
**interest earned when
you save or invest money**

Usually associated with
deposit accounts

Annual Percentage Rate (APR)

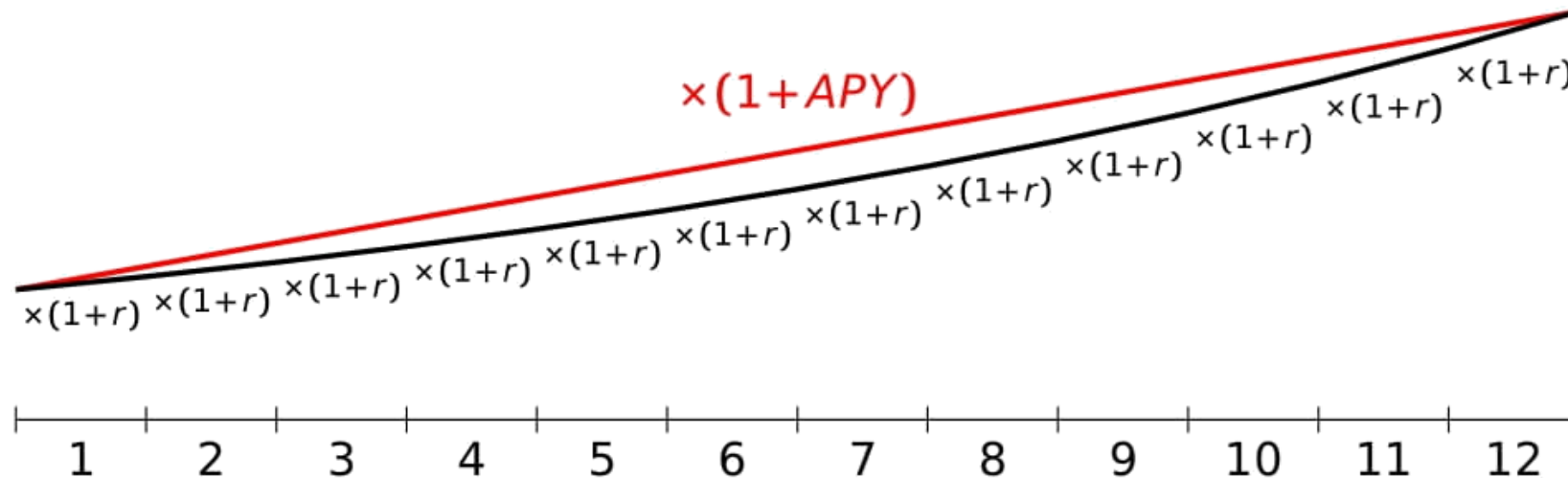
- Upošteva vse s kreditom povezane denarne tokove ($B, R_n \Rightarrow a$)
 (od glavnice odštej stroške odobritve, anuitetam prištej stroške vodenja kredita)
 (v šolskih nalogah te podrobnosti izpustimo)



- Mesečna obrestna mera r ($\tilde{B}, \tilde{a} \Rightarrow r$)
- Letna obrestna mera $\boxed{APR = 12r \Rightarrow r = \frac{APR}{12}}$
- Uporabljamo **relativni način obrestovanja**

Annual Percentage Yield (APY)

- Ne upošteva stroškov
- Upošteva učinek obrestnega obrestovanja



- Mesečna obrestna mera r
- Letna obrestna mera $1 + APY = (1 + r)^{12} \Rightarrow r = \sqrt[12]{1 + APY} - 1$
- Uporabljamo **konformni način obrestovanja**

(a) (5 pt) Congratulations! You are admitted to Harvard and approved for a student loan to finance tuition (\$60,000 per year), room and board (\$20,000 per year), and other expenses (\$7,000 per year) for four years. The loan has an annual interest rate of 8% and must be repaid in equal annual payments over 30 years after graduation (assume there is no interest while you are in college, the first payment happens in 5 years from now).

Explain why your annual payments are \$30,912.

(e) (10 pt) To pay for the education, the government decided to increase income tax to 50% for the high-income earners (starting at \$100k a year). Unfortunately, higher taxes have reduced the expected economic growth from 3% to 1% a year. Assume that a typical working span is 45 years and calculate the value of after-tax salaries under the educational loans (starting salary \$100k, grows at 3% for 45 years, tax 25%, minus the loan repayments) and then compare it with the government-funded education (starting salary \$100k, grows at 1% for 45 years, tax 50%, no loan repayments). Discount at 8%. Explain which one is better for you.

(a) The Individual Burden (5 points)

Calculation Annual Payment:

Loan Amount = $(\$60,000 + \$20,000 + \$7,000) * 4 = \$348,000$

Plugging these into the **annuity formula**: $0.08 * \$348,000 / (1 - 1/1.08^{30}) = \$\mathbf{30,912}$

(e) Paying for the government-funded education (10 points)

Scenario 1: Education Loans

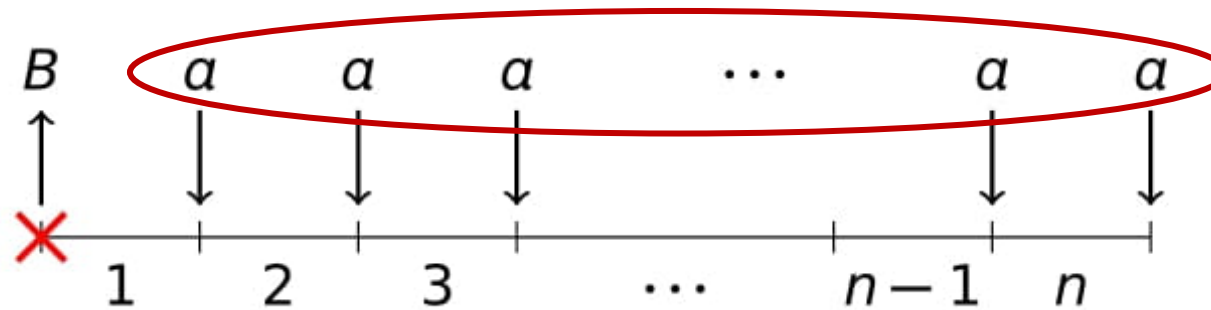
Using the **growing annuity formula**, the **present value** of the after-tax salary payments in the first scenario:

$$1) \quad PV(\text{after} - \text{tax salary}) = \frac{\$100,000 * (1 - 0.25)}{0.08 - 0.03} \left(1 - \frac{(1 + 0.03)^{45}}{(1 + 0.08)^{45}} \right) = \$1,322,300$$

Since the discount rate is the same as the interest rate on the loan, the present value of the loan payments is equal to the value of the loan. Hence, the net value of this option to you is $\$1,322,300 - \$348,000 = \$974,300$.

Annuity Formula

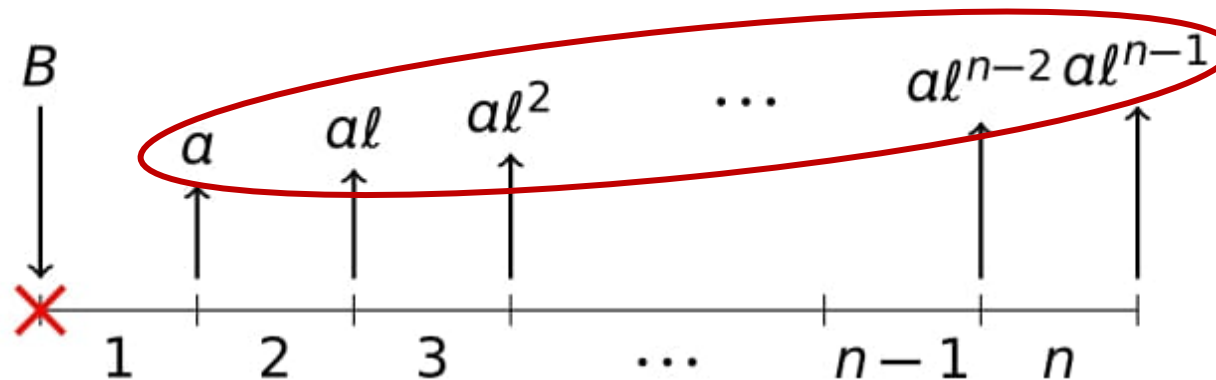
- Sedanja vrednost postnumerandne rente
- Višina anuitete kredita z glavnico B
- Obdobni obrestni faktor $k = 1 + r$



$$\begin{aligned}
 & \bullet B = \frac{a}{k} + \frac{a}{k^2} + \dots + \frac{a}{k^n} = \frac{a}{k^n} (k^{n-1} + k^{n-2} + \dots + 1) = \frac{a}{k^n} \cdot \frac{k^n - 1}{k - 1} = \boxed{\frac{a}{(1+r)^n} \cdot \frac{(1+r)^n - 1}{r}} \\
 & \bullet a = Bk^n \cdot \frac{k - 1}{k^n - 1} = \boxed{Br \cdot \frac{(1+r)^n}{(1+r)^n - 1}} = \boxed{Br \cdot \frac{1}{1 - \frac{1}{(1+r)^n}}}
 \end{aligned}$$

Growing Annuity Formula

- Sedanja vrednost naraščajoče postnumerandne rente
- Obdobni obrestni faktor $k = 1 + r$
- Obdobni faktor rasti $\ell = 1 + g$



$$\bullet B = \frac{a}{k} + \frac{a\ell}{k^2} + \dots + \frac{a\ell^{n-1}}{k^n} = \frac{a}{k} \left(1 + \frac{\ell}{k} + \dots + \frac{\ell^{n-1}}{k^{n-1}} \right) = \frac{a}{k} \cdot \frac{\left(\frac{\ell}{k}\right)^n - 1}{\frac{\ell}{k} - 1}$$

$$\bullet B = \frac{a}{\ell - k} \cdot \frac{\ell^n - k^n}{k^n} = \frac{a}{r - g} \cdot \frac{(1+r)^n - (1+g)^n}{(1+r)^n} = \boxed{\frac{a}{r - g} \cdot \left(1 - \frac{(1+g)^n}{(1+r)^n} \right)}$$

Question 25. Investment, Financial Instruments, and Risk Management

Rick is a young devoted father and wants to **save up to finance** his daughter Collete's college expenses. The cost of four years of college right now is \$250,000 and it is projected to grow at 12% a year. Colette is currently 3 years old so Rick has **15 years of investment**. He is considering the following options: high-yield **savings account** that delivers 5% a year or investing in a **mutual fund** that replicates S&P 500 returns with annual return on average of 10% a year. What is the **annual investment** that Rick needs to do under each one of the investment plans: (annuity factor for 15 years at 5% is 21.58, at 10% is 31.77)

- A. Savings: \$63,410, Investment: \$43,071
- B. Savings: \$53,410, Investment: \$53,410
- C. Savings: \$60,000, Investment: \$30,000
- D. Savings: \$61,315, Investment: \$40,851

CASIO ELECTRONIC CALCULATOR FC-100V

FINANCIAL CONSULTANT

Cash Flow

I_x = 3

Cash=0, Editor x

NPV: Solve



Present Value Annuity Tables

$$\text{Formula: } PV = [1 - 1 / (1 + i)^n] / i$$

n / i		5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%
1		0.924	0.9434	0.9346	0.9259	0.9174	0.9091	0.9009	0.8929	0.8850	0.8772	0.8696
2		1.794	1.8334	1.8080	1.7833	1.7591	1.7355	1.7125	1.6901	1.6681	1.6467	1.6257
3		2.632	2.6730	2.6243	2.5771	2.5313	2.4869	2.4437	2.4018	2.3612	2.3216	2.2832
4		3.460	3.4651	3.3872	3.3121	3.2397	3.1699	3.1024	3.0373	2.9745	2.9137	2.8550
5		4.295	4.2124	4.1002	3.9927	3.8897	3.7908	3.6959	3.6048	3.5172	3.4331	3.3522
6		5.075	4.9173	4.7665	4.6229	4.4859	4.3553	4.2305	4.1114	3.9975	3.8887	3.7845
7		5.864	5.5824	5.3893	5.2064	5.0330	4.8684	4.7122	4.5638	4.4226	4.2883	4.1604
8		6.632	6.2098	5.9713	5.7466	5.5348	5.3349	5.1461	4.9676	4.7988	4.6389	4.4873
9		7.408	6.8017	6.5152	6.2469	5.9952	5.7590	5.5370	5.3282	5.1317	4.9464	4.7716
10		8.117	7.3601	7.0236	6.7101	6.4177	6.1446	5.8892	5.6502	5.4262	5.2161	5.0188

Future Value Annuity Tables

$$\text{Formula: } FV = [(1 + i)^n - 1] / i$$

n / i		6%	7%	8%	9%	10%	11%	12%	13%	14%	15%
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0600	2.0700	2.0800	2.0900	2.1000	2.1100	2.1200	2.1300	2.1400	2.1500	2.1600
3	3.1836	3.2149	3.2464	3.2781	3.3100	3.3421	3.3744	3.4069	3.4396	3.4725	3.5056
4	4.3746	4.4399	4.5061	4.5731	4.6410	4.7097	4.7793	4.8498	4.9211	4.9934	5.0668
5	5.6371	5.7507	5.8666	5.9847	6.1051	6.2278	6.3528	6.4803	6.6101	6.7424	6.8772
6	6.9753	7.1533	7.3359	7.5233	7.7156	7.9129	8.1152	8.3227	8.5355	8.7537	8.9772
7	8.3938	8.6540	8.9228	9.2004	9.4872	9.7833	10.0890	10.4047	10.7305	11.0668	11.4137
8	9.8975	10.2598	10.6366	11.0285	11.4359	11.8594	12.2997	12.7573	13.2328	13.7268	14.2393
9	11.4913	11.9780	12.4876	13.0210	13.5795	14.1640	14.7757	15.4157	16.0853	16.7858	17.5083
10	13.1808	13.8164	14.4866	15.1929	15.9374	16.7220	17.5487	18.4197	19.3373	20.3037	21.3201

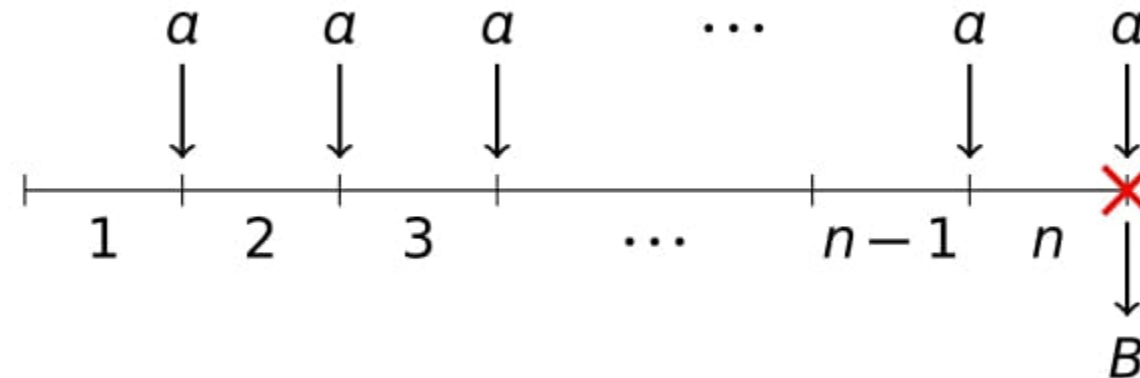
Future Value Annuity Tables

$$\text{Formula: } FV = [(1 + i)^n - 1] / i$$

n / i	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0100	2.0200	2.0300	2.0400	2.0500	2.0600	2.0700	2.0800	2.0900	2.1000	2.1100	2.1200	2.1300	2.1400	2.1500
3	3.0301	3.0604	3.0909	3.1216	3.1525	3.1836	3.2149	3.2464	3.2781	3.3100	3.3421	3.3744	3.4069	3.4396	3.4725
4	4.0604	4.1216	4.1836	4.2465	4.3101	4.3746	4.4399	4.5061	4.5731	4.6410	4.7097	4.7793	4.8498	4.9211	4.9934
5	5.1010	5.2040	5.3091	5.4163	5.5256	5.6371	5.7507	5.8666	5.9847	6.1051	6.2278	6.3528	6.4803	6.6101	6.7424
6	6.1520	6.3081	6.4684	6.6330	6.8019	6.9753	7.1533	7.3359	7.5233	7.7156	7.9129	8.1152	8.3227	8.5355	8.7537
7	7.2135	7.4343	7.6625	7.8983	8.1420	8.3938	8.6540	8.9228	9.2004	9.4872	9.7833	10.0890	10.4047	10.7305	11.0668
8	8.2857	8.5830	8.8923	9.2142	9.5491	9.8975	10.2598	10.6366	11.0285	11.4359	11.8594	12.2997	12.7573	13.2328	13.7268
9	9.3685	9.7546	10.1591	10.5828	11.0266	11.4913	11.9780	12.4876	13.0210	13.5795	14.1640	14.7757	15.4157	16.0853	16.7858
10	10.4622	10.9497	11.4639	12.0061	12.5779	13.1808	13.8164	14.4866	15.1929	15.9374	16.7220	17.5487	18.4197	19.3373	20.3037
11	11.5668	12.1687	12.8078	13.4864	14.2068	14.9716	15.7836	16.6455	17.5603	18.5312	19.5614	20.6546	21.8143	23.0445	24.3493
12	12.6825	13.4121	14.1920	15.0258	15.9171	16.8699	17.8885	18.9771	20.1407	21.3843	22.7132	24.1331	25.6502	27.2707	29.0017
13	13.8093	14.6803	15.6178	16.6268	17.7130	18.8821	20.1406	21.4953	22.9534	24.5227	26.2116	28.0291	29.9847	32.0887	34.3519
14	14.9474	15.9739	17.0863	18.2919	19.5986	21.0151	22.5505	24.2149	26.0192	27.9750	30.0949	32.3926	34.8827	37.5811	40.5047
15	16.0969	17.2934	18.5989	20.0236	21.5786	23.2760	25.1290	27.1521	29.3609	31.7725	34.4054	37.2797	40.4175	43.8424	47.5804
16	17.2579	18.6393	20.1569	21.8245	23.6575	25.6725	27.8881	30.3243	33.0034	35.9497	39.1899	42.7533	46.6717	50.9804	55.7175
17	18.4304	20.0121	21.7616	23.6975	25.8404	28.2129	30.8402	33.7502	36.9737	40.5447	44.5008	48.8837	53.7391	59.1176	65.0751
18	19.6147	21.4123	23.4144	25.6454	28.1324	30.9057	33.9990	37.4502	41.3013	45.5992	50.3959	55.7497	61.7251	68.3941	75.8364
19	20.8109	22.8406	25.1169	27.6712	30.5390	33.7600	37.3790	41.4463	46.0185	51.1591	56.9395	63.4397	70.7494	78.9692	88.2118
20	22.0190	24.2974	26.8704	29.7781	33.0660	36.7856	40.9955	45.7620	51.1601	57.2750	64.2028	72.0524	80.9468	91.0249	102.4436
21	23.2392	25.7833	28.6765	31.9692	35.7193	39.9927	44.8652	50.4229	56.7645	64.0025	72.2651	81.6987	92.4699	104.7684	118.8101
22	24.4716	27.2990	30.5368	34.2480	38.5052	43.3923	49.0057	55.4568	62.8733	71.4027	81.2143	92.5026	105.4910	120.4360	137.6316
23	25.7163	28.8450	32.4529	36.6179	41.4305	46.9958	53.4361	60.8933	69.5319	79.5430	91.1479	104.6029	120.2048	138.2970	159.2764
24	26.9735	30.4219	34.4265	39.0826	44.5020	50.8156	58.1767	66.7648	76.7898	88.4973	102.1742	118.1552	136.8315	158.6586	184.1678
25	28.2432	32.0303	36.4593	41.6459	47.7271	54.8645	63.2490	73.1059	84.7009	98.3471	114.4133	133.3339	155.6196	181.8708	212.7930

Future Value Annuity Factor

- Prihodnja vrednost postnumerandne rente
- Obdobni obrestni faktor $k = 1 + r$



$$B = ak^{n-1} + ak^{n-2} + \dots + a = a(k^{n-1} + k^{n-2} + \dots + 1) = a \cdot \frac{k^n - 1}{k - 1} = a \cdot \frac{(1+r)^n - 1}{r}$$

$$\boxed{FVAF = \frac{(1+r)^n - 1}{r}}$$

$$r = 5\% \text{ in } n = 15 \Rightarrow FVAF = \frac{1,05^{15} - 1}{0,05} = 21,5786$$

Zaključek

- Izrazi in oznake v finančni matematiki so rezultat tradicije, zakonodaje in priporočil, ki veljajo v posamezni državi.
- Z vidika finančne pismenosti je koristno poznati izraze in oznake, ki so pogosti v mednarodnem okolju (v angleščini so referenca ZDA).
- Pomen medpredmetnega povezovanja na Ekonomski fakulteti (Berk, J. B., & DeMarzo, P. M. (2014). *Corporate Finance*. Pearson Education)
- Kar nekaj dodatnih vsebin, ki jih bomo morali vključiti v priprave na Mednarodno ekonomsko olimpijado

Literatura

- Banka Slovenije (2008). *Priporočila o načinu obračunavanja obresti za posle s prebivalstvom* (spremno besedilo: J. A. Čibej). Združenje bank Slovenije in Banka Slovenije.
- Berk, J. B., & DeMarzo, P. M. (2014). *Corporate Finance* (3. izdaja). Pearson Education.
- Čibej, J. A. (2011). *Kako računati obresti*. Združenje bank Slovenije.
- Kokol Bukovšek, D., & Ferbar Tratar, L. (2019). *Matematika za poslovne in ekonomske vede*. Univerza v Ljubljani, Ekonomska fakulteta.
- Predmetna komisija (A. Žakelj in drugi) (2008). *Učni načrt: matematika; Gimnazija: splošna, klasična in strokovna gimnazija*. Ministrstvo za šolstvo in šport in Zavod RS za šolstvo.
- Toman, A. (2019). *Mathematics for Business and Economics: Theory Review and Problems*. Univerza v Ljubljani, Ekonomska fakulteta.
- <https://www.double-entry-bookkeeping.com/tag/tvm-table/>
- <https://www.capitalone.com/learn-grow/money-management/apr-vs-apy/>
- <https://www.casio.com/intl/scientific-calculators/product.FC-100V/>

Hvala za pozornost

ales.toman@ef.uni-lj.si