The Book of Abstracts, Complex Analysis, Geometry, and Dynamics IV.-Portorož 2025

June 3, 2025

TIMETABLE

	Monday	Tuesday	Wednesday	Thursday	Friday
9:00-10:00	**Trapani**	**Popovici**	**Roeder**	**Cheng**	**X. Huang**
	Short break	Short break	Short break	Short break	Short break
10:10-11:10	**Savale**	**Evdoridou**	**Jové**	**Takeuchi**	**Deng**
	Coffee break	Coffee break	Coffee break	Coffee break	Coffee break
11:30-12:30	**G. Huang**	**Ruggiero**	**Yum**	**Hsiao**	**Rong**
					Short break
12:40-13:40	lunch	lunch		lunch	**Zhou**
			Free afternoon		
15:00-16:00	**Palmisano**	**Ivashkovych**		**Bisi**	
	Coffee break	Coffee break		Coffee break	
16:20-17:20	**Martí-Pete**	**Kosinski**		**Bharali**	
20:00			**Conference dinner**		

A metrical notion of negative curvature and applications to the Kobayashi metric

Gautam Bharali

Indian Institute of Science, Bangalore

Visibility with respect to the Kobayashi distance is a notion that, in recent years, has had many applications in realms where one needs to control the behaviour of a given class of holomorphic maps into domains with the visibility property. Loosely speaking, this property — for a domain Ω in a complex manifold — is that all geodesics with end-points close to two distinct points in $\partial\Omega$ must bend uniformly into Ω. This is a weak notion of negative curvature (which, in fact, is a consequence of negative sectional curvature in the Riemannian category). This notion is not intrinsic since it involves the topological boundary, as opposed to some metrical boundary. A good part of this talk will be devoted to why, despite this defect, visibility has proved to be curiously effective. We shall see some conditions for a domain to be a visibility space in the sense alluded to above, keeping in mind the fact that (Ω, K_Ω), K_Ω being the Kobayashi distance, is in general not a geodesic space. We shall look at a few applications. If time permits, we shall discuss the connections between visibility and Gromov hyperbolicity of (Ω, K_Ω).

Invariants and automorphisms for slice regular functions: from some Clifford algebras, as the quaternions, to the octonions

Cinzia Bisi

Ferrara University, Italy

Let A be one of the following Clifford algebras: $\mathbb{R}_2 \cong \mathbb{H}, \mathbb{R}_3$, or $A = \mathbb{O}$. For the algebra A, the automorphism group Aut(A) and its invariants are well known. In this talk, I will describe the invariants of the automorphism group of the algebra of slice regular functions over A. All the results can be found in : arXiv:2411.16762 arXiv:2411.15896. This is joint work with Joerg Winkelmann.

Positive mass theorem and the Yamabe equation on CR manifolds

Jih-Hsin Cheng
Institute of Mathematics, Academia Sinica, Taipei

In this survey talk, I would introduce the notion of the mass in several complex variables or CR geometry. We then consider the Yamabe problem on CR manifolds to find a minimizer for the CR Sobolev quotient. The positive mass theorem plays a key role in finding a solution to the Yamabe equation with minimum energy for the positive curvature case. In this line of research I would sketch the ideas of the proofs for different dimensions in the team works of Hung-Lin Chiu, Andrea Malchiodi, Paul Yang and myself, and also related works of Chin-Yu Hsiao, Po-Lam Yung and Yuya Takeuchi.

Residual Monge-Ampère measure of plurisubharmonic functions and the zero mass conjecture

Fusheng Deng
University of Chinese Academy of Sciences

For plurisubharmonic functions φ and ψ lying in the Cegrell class of $\mathbb{B}^n$ and $\mathbb{B}^m$ respectively such that the Lelong number of φ at the origin vanishes, we show that the mass of the origin with respect to the measure $(dd^c \max\{\varphi(z), \psi(Az)\})^n$ on $\mathbb{C}^n$ is zero for $A \in \mathrm{Hom}(\mathbb{C}^n, \mathbb{C}^m) = \mathbb{C}^{nm}$ outside a pluripolar set. For a plurisubharmonic function φ defined in a neighborhood of the origin in $\mathbb{C}^n$, we introduce a notion called the *truncate threshold* of φ at 0, which encodes the growth property of φ near the origin and is denoted by $lt(\varphi, x)$, and derive an optimal estimate of the residual Monge-Ampère mass of φ at 0 in terms of its higher order Lelong numbers $\nu_j(\varphi)$ at 0 for $1 \leq j \leq n-1$, in the case that $lt(\varphi, x) < \infty$. These results provide a partial solution to the zero mass conjecture of Guedj and Rashkovskii, and improve some results connected to this conjecture in the literatures. This lecture is based on a joint work with Yinji Li, Qunhuan Liu, Zhiwei Wang, and Xiangyu Zhou.

Contracting compositions of inner functions

Vasiliki Evdoridou
The Open University

In the setting of iteration of inner functions, Aaronson and also Doering and Mañ é gave a remarkable dichotomy about the behaviour of the boundary extension. They showed that this behaviour is of one of two entirely different types, depending on the size of the sequence $(|f^n(0)|)$. We will discuss the two parts of this dichotomy in the setting of forward compositions of inner functions. We will focus on the second part, which is more intriguing, and present some recent results in the case where the composition is contracting. This is joint work with A.M. Benini, N. Fagella, P. Rippon and G. Stallard.

Semi-classical Bergman kernel asymptotics on weakly pseudoconvex domains

Chin-Yu Hsiao

Department of Mathematics, National Taiwan University, Taiwan

In this talk, I will report my recent works about Bergman kernel asymptotics on weakly pseudoconvex domains. This talk is based on joint work with George Marinescu, Xiaoshan Li and Guokian Shao

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Carleman Approximation of Symplectic Diffeomorphisms for Calogero-Moser spaces

Gaofeng Huang

University of Bern

A classical theorem of Torsten Carleman in 1927 states that any continuous function on the real axis can be approximated by entire functions on the real line. For diffeomorphisms of a totally real submanifold, Carleman approximation means to approximate these by biholomorphisms of the complex manifold on the real submanifold. Kutzschebauch and Wold considered the case $\mathbb{R}^s \subset \mathbb{C}^n$ where $s < n$. Later, Deng and Wold worked out the symplectic case $\mathbb{R}^{2n} \subset \mathbb{C}^{2n}$ and generalized it to the coadjoint orbits of complex Lie groups. A key ingredient in the proof is the symplectic density property of $\mathbb{C}^{2n}$, which says that the Lie algebra generated by globally integrable holomorphic symplectic vector fields is dense in all holomorphic symplectic fields. Another Stein symplectic manifold with this property is the so-called Calogero-Moser (CM) space. We will see how this leads to a Carleman approximation of symplectic diffeomorphisms of the real CM space, given by the fixed point set of an involution, by symplectic biholomorphisms of the complex CM space.

Bounding a Levi-flat Hypersurface in a Stein Manifold

Xiaojun Huang

Rutgers University

Let M be a smooth real codimension two compact submanifold in a Stein manifold. We will discuss the following theorem: Suppose that M has two elliptic complex tangents and that CR points are non-minimal. Assume further that M is contained in a bounded strongly pseudoconvex domain. Then M bounds a unique smoothly up to M Levi-flat hypersurface $\widehat{M}$ that is foliated by Stein hyper-surfaces diffeomorphic to the ball. Moreover, $\widehat{M}$ is the hull of holomorphy of M . This subject has a long history of investigation dating back to E. Bishop and Harvey-Lawson. I will discuss both the historical context and the techniques used in the proof of the aforementioned theorem.

This is from a joint work with Hanlong Fang, Wanke Yin and Zhengyi Zhou.

Positivity of the Boundary Intersections of J-complex Curves and Adjunction Formula

Serge Ivashkovych
Lille University

Let an almost complex structure J in a neighborhood B of the origin in $\mathbb{R}^4$ be given and let $0 \in W \subset B$ be a germ of a J-totally real surface. After an appropriate coordinate change, (called *a redressing map*), we can assume wlog that $(B, W) = (\mathbb{R}^4, \mathbb{R}^2)$ and $J|_{\mathbb{R}^2} = J_{st}$. Denote by $\Delta^+ := \{z \in \Delta : \mathsf{Im}\, \mathsf{z} \geq 0\}$ the upper half-disk and by $\beta_0 := (-1, 1)$ its edge. Let a J-holomorphic map $u : (\Delta^+, \beta_0, 0) \to (\mathbb{R}^4, \mathbb{R}^2, 0)$ be given, assume it is continuous up to β_0. We shall call $M^+ := u(\Delta^+)$ a J-complex half-disk attached to W. Making a reflection with respect to $W = \mathbb{R}^2$ we can extend u to Δ with an appropriate regularity. In the standard complex coordinates of $\mathbb{C}^2 = (\mathbb{R}^4, J_{st})$ our extension $\tilde{u}$ writes as

$$\tilde{u}(z) = \mathrm{v}_0 z^\mu + O(|z|^{\mu+\alpha}) \quad \text{with} \quad \text{some} \quad \mu \in \mathbb{N} \quad \text{and} \quad \mathrm{v}_0 \neq 0 \in \mathbb{R}^2.$$

We shall call v_0 the *tangent vector* to M^+ at zero and the number μ the order of vanishing of u (and of $\tilde{u}$) at zero. This μ doesn't depend on the redressing map. Now let $u_1, u_2 : (\Delta^+, \beta_0, 0) \to (\mathbb{R}^4, \mathbb{R}^2, 0)$ be two J-complex half-disks attached to W. We say that u_1 is a reparameterization of u_2 is there exists a holomorphic function $\psi(\zeta) = \zeta + O(\zeta^2)$, real for real ζ, such that in a neighborhood of the origin in Δ^+ one has

$$u_1(\zeta) = u_2(\psi(\zeta)).$$

We define the *boundary intersection index* of u_1 and u_2 at zero as the intesection number at zero of their extensions by reflection, i.e., of $\tilde{u}_1$ and $\tilde{u}_2$. Denote this index as $\mathbf{ind}^{\mathbf{b}}_{\mathbf{0}}(u_1, u_2)$. Our goal is to present the following statements.

Theorem 1. *Assume that our half-disks are such that one is not a reparameterization of another. Then their boundary intersection index is correctly defined, doesn't depend on the redressing map, and satisfies*

$$\mathbf{ind}^{\mathbf{b}}_{\mathbf{0}}(u_1, u_2) \geq \mu_1 \cdot \mu_2,$$

where μ_i is the order of vanishing of u_i at zero, $i = 1, 2$. In particular $\mathbf{ind}^{\mathbf{b}}_{\mathbf{o}}$ is positive.

Let C be a compact Riemann surface of genus g with boundary $\dot{\mathrm{C}}$ consisting from σ circles and $M = u(\bar{C})$ its image under a J-holomorphic map $u : \bar{C} \to (X, J)$ such that $\dot{\mathrm{M}} := u(\dot{\mathrm{C}}) \subset W$, here W is a totally real surface in an almost complex surface (X, J).

Theorem 2. *(Adjunction Formula) Assume that M has no cusps on W. Then*

$$2g + \sigma = \frac{\mu_{\dot{M}}(X, W) - [M^d]^2}{2} + 2 - \delta^{(b)} - 2\delta^{(i)} - 2\kappa^{(i)},$$

where $\delta^{(b)}$ (corr. $\delta^{(i)}$) is the sum of all boundary (corr. inner) intersection indices of the self-intersection points of M and $\kappa^{(i)}$ is the sum of the cusp indices of all (inner) cusps of M.

Here μ is the Maslov index of W along ∂M and M^d is the Shottky double of M. The self-intersection $[M^d]^2$ is to be explained during the talk. To my best knowledge these results are new even for integrable J-s unless W is supposed to be real-alytic.

Now consider two *analytic* half-disks M_1, M_2 in $\mathbb{C}^2$, i.e., M_k is the image of a holomorphic in the standard sense embedding $u_k : \Delta^+ \to \mathbb{C}^2$ smooth up to $\partial_0\Delta^+$ and such that $u_1(0) = u_2(0) = 0$. We prove the following statement:

Theorem 3. *Suppose that the order of tangency of M_1 with M_2 at zero is finite and equal to d. Then the index of intersection of M_1 with M_2 at zero is well defined and is equal to d. This is index is equal to 1 if and only if M_1 intersects M_2 at zero tranversely.*

The proof of this theorem will be achieved through the construction of a totally real surface $W \ni 0$ such that M_1 and M_2 are attached to W and then the index in question is nothing but the boundary intersection index $\mathbf{ind}^{\mathbf{b}}_{\mathbf{p_0}}(u_1, u_2)$ as in Theorem 1.

Boundaries of multiply connected Fatou components. A unified approach

Anna Jové

Universitat de Barcelona

In this talk, we will focus on boundaries of multiply connected Fatou components, from a topological, measure-theoretical and dynamical point of view. The main tool in our analysis is the universal covering map (and its boundary extension), which allows us to relate the dynamics on the boundary with the dynamics of the radial extension of the so-called associated inner function. This way, we can deal with all Fatou components (invariant or wandering, with all possible internal dynamics) simultaneously. We will explore the similarities and the differences that appear for Fatou components of transcendental functions (both invariant and wandering) in contrast with rational maps. This is joint work with G. R. Ferreira.

TBA

Łukasz Kosiński

Jagiellonian University

Wandering dynamics of transcendental functions

David Marti-Pete
University of Liverpool

In the recent years there have been a lot of new constructions of transcendental functions with wandering domains, following the work of Boc Thaler in 2021. Together with Rempe and Waterman, we proved that every full compact set is a wandering compact set for some transcendental entire function. On the other hand, Benini, Evdoridou, Fagella, Rippon and Stallard classified the internal dynamics of wandering domains and also constructed several examples of each type. In my talk I will give an overview of these constructions and discuss a new and more general way of using approximation theory to obtain transcendental functions with wandering continua which allows us to prescribe the internal dynamics; more precisely, the resulting function is conjugate on the closure of a wandering domain to the model map used in the approximation. This is a joint work with Vasiliki Evdoridou and Lasse Rempe.

Stability of non-hyperbolic sets

Liviana Palmisano
KTH Royal Institute of Technology

Hyperbolic sets are know to be structurally stable, namely they remain unchanged after perturbations in open neighborhoods. The aim of the talk is to discuss the case on certain non-hyperbolic sets in two-dimension. Do they have some form of stability? Given a two dimensional unfolding of a map with a homoclinic tangency, we prove the existence a codimension one lamination of maps with a non-hyperbolic set whose topology persists along the leaves of the lamination. Stability of other non-hyperbolic sets in higher dimensional families will also be discussed.

Higher-Degree Holomorphic Contact Structures

Dan Popovici
University of Toulouse, France

This is joint work with H. Kasuya and L. Ugarte.

We introduce the classes of holomorphic p-contact manifolds and holomorphic s-symplectic manifolds that generalise the classical holomorphic contact and holomorphic symplectic structures. After observing their basic properties and exhibiting a wide range of examples, we give three types of general conceptual results involving the former class of manifolds: structure theorems; hyperbolicity results; unobstructedness theorems, generalising to our context the classical Bogomolov-Tian-Todorov theorem, for two types of small deformations of complex structures that generalise the small essential deformations previously introduced for the Iwasawa manifold and for Calabi-Yau page-1-$\partial\bar{\partial}$-manifolds.

Dynamics of toric rational maps

Roland Roeder
Indiana University Indianapolis

I will describe a class of rational maps in two complex variables that preserve the meromorphic two form $\eta = \frac{dx \wedge dy}{xy}$. This property makes their dynamics easier to study, while still providing rich examples. Indeed, the mappings that were recently proved by Bell-Diller-Jonsson to have transcendental dynamical degree preserve η. Such mappings do not admit algebraically stable models. In this talk I will explain my joint work with Jeffrey Diller investigating the equidistribution and ergodic properties of these mysterious mappings.

Some boundary rigidity results for holomorphic self-maps of bounded domains

Feng Rong
Shanghai Jiao Tong University

We present some of our recent boundary rigidity results for holomorphic self-maps of bounded domains. The first part focuses on the Burns-Krantz rigidity and its variants, while the second part is devoted to the n-point boundary rigidity.

On the Dynamical Manin-Mumford problem for plane polynomial endomorphisms.

Matteo Ruggiero
Université Paris Cité - IMJ-PRG

The Dynamical Manin-Mumford problem is a dynamical question inspired by classical results from arithmetic geometry. Given an algebraic dynamical system (X, f), where X is a projective variety and f is a polarized endomorphism on X, we want to determine if a subvariety Y containing "unusually many" periodic points must be itself preperiodic. In a work in collaboration with Romain Dujardin and Charles Favre, we prove this property to hold when f is a regular endomorphism of $\mathbb{P}^2$ coming from a polynomial endomorphism of $\mathbb{C}^2$ of degree $d \geq 2$, under the additional condition that the action of f at the line at infinity doesn't have periodic superattracting points. The proof is an interesting blend of techniques coming from arithmetic geometry, holomorphic and non-archimedean dynamics.

Bochner Laplacians and Bergman kernels for families

Nikhil Savale
Universität zu Köln

We generalize the results of Marinescu-Savale to families of Bochner Laplacians. This particularly leads to the fiberwise expansion for families Bergman kernels of horizontally semi-positive index bundles. Our results include the case of highest weight families. The proofs are based on Ma-Zhang's description for the curvature of the index bundle as a fiberwise Toeplitz operator as well as on methods from sub-Riemannian geometry. Based on joint work with X. Ma and G. Marinescu.

CR Paneitz operator and embeddability

Yuya Takeuchi
University of Tsukuba

The CR Paneitz operator, a CR invariant fourth-order linear differential operator, plays a crucial role in three-dimensional CR geometry. It is deeply connected to global embeddability, the CR positive mass theorem, and the logarithmic singularity of the Szegő kernel. In this talk, I will discuss recent progress on the spectrum of the CR Paneitz operator. Specifically, I will focus on differences in its nature depending on whether it is embeddable or not.

Various forms of weak Gromov Kähler hyperbolicity

Stefano Trapani
Dipartimento di Matematica Universita' di Roma Tor Vergata, Roma Italy

I will describe the content of three papers written in collaboration with Francesco Bei, Simone Diverio, Benoit Claudon, and Phillip Eyssidiex concerning different weak forms of Gromov Kähler hyperbolicity. The notion of Kähler hyperbolic manifolds was introduced by Michael Gromov in 1991, he showed many non trivial properties of such manifolds concerning in particular their Kobayashy hyperbolicity and spectral properties of the laplacian acting on$L^2(p,q)$ forms on their universal covering. Our first paper concerns a weak version of Kälher hyperbolicity which maintains many properties of the original Kähler hyperblocity and has interesting connection with Green-Griffith and Lang-conjecture. In the second paper we proved that such notion of weak Kähler hyperbolicity is invariant by birational maps answering a question by Janosh Kolar. Finally in the third paper we introduced an even weaker notion which nevertheless enjoys some nice properties, for example the existence of non trivial holomorphic sections on Nakano positive vector bundles, and some curvature bounds.

Statistical Bergman Geometry and Schwarz lemma

Jihun Yum
Jeonbuk National University

This presentation investigates the Bergman geometry of bounded domains Ω in $\mathbb{C}^n$ through the lens of Information geometry by introducing an embedding $\Phi : \Omega \to \mathcal{P}(\Omega)$, where $\mathcal{P}(\Omega)$ denotes a space of probability distributions on Ω. A result by J. Burbea and C. Rao establishes that the pullback of the Fisher information metric, the fundamental Riemannian metric in Information geometry, via Φ coincides with the Bergman metric of Ω. Building on this idea, we consider Ω as a statistical model in $\mathcal{P}(\Omega)$ and present several interesting results within this framework. In particular, we focus on a Schwarz-type lemma for the Bergman metric.

The Schwarz-Pick lemma in one complex variable states that for any holomorphic function $f : \mathbb{D} \to \mathbb{D}$ between two unit discs,

$$f^* g_{\mathbb{D}} \leq g_{\mathbb{D}},$$

where $g_{\mathbb{D}}$ denotes the Bergman metric of the unit disc $\mathbb{D}$ in $\mathbb{C}$. Adopting a statistical viewpoint, we extend this inequality to bounded domains in $\mathbb{C}^n$, under additional conditions on the domain. We also discuss how this result compares with Yau's Schwarz lemma for Kähler metrics.

$\overline{\partial}$ equations with L^2 estimates and bundles with singular positive curvature

Xiangyu Zhou
Institute of Mathematics, Chinese Academy of Sciences

In this talk, we first recall some basic properties of multiplier ideal sheaves associated to pseudoeffective line bundles (or psh functions), including Guan-Zhou's solution of Demailly's strong openness conjecture, then present Deng-Ning-Wang-Zhou's characterization of Nakano positivity via solving $\overline{\partial}$- equations with L^2 estimates, which is a converse proposition of Hörmander-Demailly's L^2 existence theorems. This gives a connection between differential geometry and differential equation via several complex variables. As an application of the characterization, we give an affirmative answer to Lempert's problem (Liu-Yang-Zhou), which asks whether the limit metric of an increasing sequence of hermitian metrics with Nakano semi positive curvature on holomorphic vector bundles is still Nakano semi-positive. As another application, one may define singular metric of positive curvature in the sense of Nakano on holomorphic vector bundles. Finally, we present recent results on the strong openness of the multiplier submodule sheaves (vector bundle version of multiplier ideal sheaves) by Liu-Xiao-Yang- Zhou, and Le Potier type isomorphism theorem between cohomology of the vector bundles twisted with the multiplier submodule sheaves and cohomology of the associated line bundles twisted with the multiplier ideal sheaves (Liu-Liu-Yang-Zhou).
