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A Julia-Wolff-Carathéodory Theorem in convex domains of finite type

Leandro Arosio
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The classical Julia-Wolff-Carathéodory shows that, if f is a holomorphic self-map of the disc, the derivative f' admits a positive nontangential limit near any boundary regular fixed point z, and the limit equals the dilation of f at z which can be computed in terms of the Poincaré distance. This result had several generalizations to several variables: in particular Rudin proved a version of it in the ball, Abate in strongly convex domains, and Abate-Tauraso in convex domains of D'Angelo finite type, adding a couple of technical assumptions. In this talk I will show how to prove the full theorem in the context of convex domains of D'Angelo finite type, using the strong asymptoticity of complex geodesics and the existence of horospheres. This result turns out to be related to the pluricomplex Poisson kernel introduced by Bracci-Patrizio-Trapani. This is a joint work with Matteo Fiacchi.

On non-autonomous attracting basins

Sayani Bera
Indian Association for the Cultivation of Science, Kolkata

The goal of this talk is to discuss briefly the idea of the proof of the Bedford's conjecture (formulated by Fornæss-Stensønes in 2004), on uniform non-autonomous attracting basins of automorphisms of $\mathbb{C}^k, k \geq 2$ and Fatou-Bieberbach domains.

Thus (as a consequence of Fornæss-Stensønes) we affirmatively answer Bedford's question (2000) on uniformizations of the stable manifolds, corresponding to a hyperbolic compact invariant subset of a complex manifold.

This is a joint work with Dr. Kaushal Verma.

The Schwarz Lemma in Kähler and Non-Kähler Geometry

Kyle Broder
The University of Queensland

A folklore conjecture generalizing a conjecture made by Kobayashi half a century ago predicts that a compact Kobayashi hyperbolic manifold is projective and canonically polarized. There have been a significant number of achievements in this direction recently, the most notable being what has come to be known as the Wu–Yau theorem: A compact Kähler manifold with a Kähler metric of negative holomorphic sectional curvature is projective and canonically polarized. One of the primary stumbling blocks to further developments is the Schwarz lemma in the Hermitian non-Kähler category. We will present some recent improvements on the Schwarz lemma and related results. If time permits, we will discuss a conjectural positive analog of the Wu–Yau theorem.

The CR Embedding Problem

Sean Curry
Oklahoma State University

I will discuss an approach to the local embedding problem in CR geometry via weighted Hodge theory for the $\overline{\partial}_b$-complex, twisted by a suitable line bundle. This is joint work in progress with Peter Ebenfelt.

Sharp subelliptic estimates for the $\bar{\partial}$-Neumann problem

Gian Maria Dall'Ara
INDAM & Scuola Normale Superiore (Pisa, Italy)

I will talk about joint work with S. Mongodi (Univ. Milano Bicocca) about the problem of determining the optimal gain of regularity in the $\bar{\partial}$-Neumann problem near a D'Angelo finite type point, as measured in the L^2-Sobolev scale. While the classical methods of Kohn (via multiplier ideals) and Catlin (via potential theory) are effective in establishing *some* gain of regularity, they do not seem to be suited to determining optimal results. We introduce a new technique, based on Fourier analysis, a new geometric notion of type for plurisubharmonic functions, and an uncertainty principle that I established a few years ago in a related setting, and show that it indeed allows to obtain sharp subelliptic estimates in a class of low dimensional examples.

Monge-Ampére Energies.

Eleonora Di Nezza
Sorbonne Université

Let X be a compact Kähler manifold. Given a measure μ on X, we look at the complex Monge-Ampère equation $MA(u) = \mu$. In this talk we give a criteria to ensure the existence of a solution in a weighted energy class answering a question raised by Guedj and Zeriahi in 2007. This is a joint work with Darvas and Lu.

Metric properties and geometry of domains in the complex Euclidean space

Gaussier Hervé
Université Grenoble Alpes - Institut Fourier

Invariant metrics, under the action of biholomorphic maps, are important objects in the study of domains in the complex Euclidean space ; for instance, they encode in their asymptotic behavior some of the geometric properties of domains, such as curvature. I will try to explain how some properties of such invariant metrics may characterize model domains, like the unit ball.

Positivity of Kähler-Einstein currents

Vincent Guedj

Paul Sabatier University - Toulouse III

Kähler-Einstein currents have been constructed on mildly singular projective varieties in the last decades. These are canonical Kähler forms at regular points which admit local bounded potentials near the singularities. In this lecture we study whether these currents are positive (i.e. they are Kähler current) near the singular locus. This is joint work with H.Guenancia and A.Zeriahi.

Q-prime curvatures associated with Chern classes

Kengo Hirachi

University of Tokyo

In CR geometry, the Q-prime curvature is a local invariant of Live forms (or contact forms) that satisfy Einstein equation and its integral gives a global CR invariant. In 3-dimensions, the global invariant agrees with the Burns-Epstein invariant. Recently, Yuya Takeuchi gives a generalization of Q-prime curvature. He constructed a local invariant associated with each Chern class via Cheng-Yau metric on strictly pseudoconvex domain and asymptotic analysis of its curvature form. The original Q-prime curvature corresponds to the degree 0 case. In this talk, we give another construction of generalized Q-prime curvatures by using Fefferman's ambient metric and then derive the variational formula for the global invariant given by the integral of each Q-prime curvature.

Curvature property of Bergman metrics over a complex manifold

Xiaojun Huang

Rutgers University

I will discuss the curvature property of Bergman metrics over a complex manifold. For the theorem I will discuss is when a pesudoconvex domain has constant holomorphic sectional curvature. We also present some examples to demonstrate the Bergman metric can take positive constant or can be flat along a totally geodesic submanifold. This will be a joint work with Song-Ying Li from UC Irvine.

Formal principle with convergence for rational curves

Jun-Muk HWANG
Institute for Basic Science, Center for Complex Geometry

We propose a conjecture that a general member of a bracket-generating family of rational curves in a complex manifold satisfies the formal principle with convergence, namely, any formal equivalence between such curves is convergent. If the normal bundles of the rational curves are positive, the conjecture follows from the results of Commichau-Grauert and Hirschowitz. As an evidence of the conjecture, we prove the conjecture for the case when the normal bundles are furthest from positive vector bundles among bracket-generating families, namely, when the families of rational curves are of Goursat type.

Complex Hessian equations

Sławomir Kołodziej
Jagiellonian University, Kraków

Complex Hessian equation have rather short history, but the research is very intensive in recent years. We give a background and survey main results and methods in domains of $\mathbb{C}^n$, on compact Kähler manifolds and on Hermitian manifolds. Some applications and open problems will be also presented.

Gromov ellipticity in complex analytic geometry and algebraic geometry

Yuta Kusakabe
Kyoto University

In the context of Oka theory, Gromov introduced several ellipticity conditions and established the Oka principle for maps from Stein manifolds to elliptic complex manifolds. Gromov ellipticity is the opposite of Kobayashi–Eisenman–Brody hyperbolicity, which means the existence of many dominating holomorphic maps from affine spaces. In this talk, we will first discuss the relationship between Gromov ellipticity and the Oka property in the complex analytic category, and then discuss to what extent analogous results hold in the algebraic category.

Finite jet determination of non-collapsing holomorphic maps

Bernhard Lamel

Texas A&M University at Qatar/Universität Wien

We discuss recent joint work with Nordine Mir and Guillaume Rond providing a general finite jet determination result for holomorphic maps taking a minimal real-analytic CR manifold $M \subset \mathbb{C}^N$ into a Nash set $M' \subset \mathbb{C}^{N'}$. We say that a map is non-collapsing if it does map M into the set of infinite type points in M'. The main result is that such maps are uniquely determined by a jet of some order.

A Geometric Approach to Polynomial and Rational Approximation

Kirill Lazebnik

University of North Texas

We will discuss an alternative approach to the classical theorem of Runge on polynomial and rational approximation in one complex variable, with particular attention towards understanding the behavior of the approximants off of the region where they approximate a given function. Our approach is based on the theory of quasiconformal mappings in the plane. This talk is based on joint work with Christopher Bishop.

A flower theorem in dimension two

Lorena López-Hernanz

Universidad de Alcalá

The local dynamics of a tangent to the identity biholomorphism in dimension one is described by Leau-Fatou flower theorem, that guarantees the existence of simply connected domains with 0 in the boundary, covering a punctured neighborhood of 0, in which the dynamics is either attracting or repelling and where the biholomorphism is conjugated to the unit translation. We present a two-dimensional version of this result, valid when the fixed point is a non-degenerate singular point. This is a joint work with Rudy Rosas.

Spiralling domains in dimension 2

Jasmin Raissy
Institut de Mathématiques de Bordeaux

In this talk, I will present a joint work in progress with Xavier Buff. We study the dynamics of polynomials endomorphisms of $\mathbb{C}^2$ which are tangent to the identity at a fixed point. Our goal is to show the existence of such maps for which the immediate basin of attraction of the fixed point has an infinite number of distinct invariant connected components, where the orbits converge to the fixed point without being tangent to any direction.

Exceptional hyperbolic CR-singularities and reversible parabolic diffeomorphisms of $(\mathbb{C}^2, 0)$.

Stolovitch, Laurent
CNRS-Université Côte d'Azur

In this joint work with Martin Klimes, we solve the problem of both formal and analytic classification of germs of real analytic surfaces in $\mathbb{C}^2$ with non-degenerate CR singularities of exceptional hyperbolic type, under the assumption that the surface is holomorphically flat, i.e. that it can be locally holomorphically embedded in a real hypersurface of $\mathbb{C}^2$. It happens that this relies on the study holomorphic germs of parabolic diffeomorphisms of $(\mathbb{C}^2, 0)$ that are reversed by a holomorphic reflection and posses an analytic first integral with non-degenerate critical point at the origin.

A better algorithm to solve equations than Newton's method

Tuyen Trung Truong
University of Oslo

A well known algorithm to solve equations is Newton's method. It is easy to implement and it has fast rate of convergence (when it converges!). However, it has big drawbacks: it may diverge to infinity, it may converge to a saddle point, or it may have attracting cycles with at least 2 elements (hence are not even critical points).

Is there an algorithm with the same good properties as Newton's method, but also has better convergence guarantee? The answer is Yes. This talk will describe a new method called Backtracking New Q-Newton's method, which is developed from earlier work by the speaker with collaborators from University of Paris, Torus AI, University of Oslo and ODI Medical AS. Both theoretical guarantees and experimental results are presented (in particular concerning basins of attraction for roots of functions in 1 variables, not necessarily polynomials).

Note that this algorithm is not algebra, because Curtis McMullen showed that there is no algebraic iterative dynamical system which will converge to roots of a general polynomial of degree at least 4.

Second jet determination for CR mappings

Alexander Tumanov
University of Illinois

We consider a problem whether a CR diffeomorphism of generic manifolds in complex space is uniquely determined by its finite jet at a point, which is referred to as finite jet determination. We derive the finite jet determination for CR diffeomorphisms of smooth Levi nondegenerate manifolds of arbitrary codimension from the finite dimensionality of the algebras of infinitesimal automorphisms of the corresponding quadrics. Previously, this implication was well known for real analytic manifolds. We prove a new 2-jet determination result that covers most affirmative results on this matter obtained so far.

Bergman kernel functions associated to measures supported on totally real submanifolds

Duc Viet Vu
University of Koeln

In the talk I will explain my recent joint-work with George Marinescu (Cologne) in which we prove that the Bergman kernel function associated to a smooth measure supported on a piecewise-smooth maximally totally real submanifold K in $\mathbb{C}^n$ is of polynomial growth (e.g, in dimension one, K is a finite union of transverse Jordan arcs in $\mathbb{C}$). Our bounds are sharp when K is smooth. We also discuss an application to equidistribution of zeros of random polynomials extending a result of Shiffman-Zelditch to the higher dimensional setting.

Oka properties of complements of unbounded convex sets

Erlend Fornæss Wold
University of Oslo

We will discuss results related to the following, obtained by the speaker and F. Forstnerič: Suppose that $E \subset \mathbb{C}^n$, $n \geq 2$, is a closed convex set that does not contain a real line. Then $\mathbb{C}^n \setminus E$ is an Oka domain.
