Invited talks

Pointwise vs. setwise stabilizers

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Given a (finite or infinite) permutation group G acting on a set Ω, let $\rho(G)$ denote the size of the smallest subset Δ of Ω such that the *setwise stabilizer* G_Δ is trivial, if such a subset Δ exists. A related classical concept is that of a *base* of G, i. e., a subset Φ of Ω such that the *pointwise stabilizer* $G_{(\Phi)}$ is trivial. Let $b(G)$ denote the minimum base size. Let us henceforth assume $\rho(G)$ exists and $b(G)$ is finite. Then, obviously, $b(G) \le \rho(G)$. We consider the **question,** implicit in a 2014 paper by Boutin and Imrich (El. J. Comb.), whether $\rho(G)$ *is bounded from above by some function of* $b(G)$ (regardless of the size of Ω). Boutin and Imrich gave a positive answer in the case that G is the automorphism group of a connected locally finite infinite graph and G is infinite.

Without such underlying structure, little seems to be known. Clearly, if $b(G) = 1$ then $\rho(G) = 1$. We give a positive answer in the case $b(G) = 2$. Our proof is surprisingly nontrivial; it is based on the the study of sharply arc-transitive group actions on cliques, balanced complete bipartite graphs, and cocktail party graphs. (A *cocktail party graph* is the complement of a perfect matching. We say that the group G acts *sharply arc-transitively* on the graph X if $G \le \mathrm{Aut}(X)$ and for every pair $((u, v), (x, y))$ of pairs of adjacent vertices, there is a unique permutation $\sigma \in G$ such that $\sigma(u) = x$ and $\sigma(v) = y$.) We use group theory, including results on Frobenius groups, in the finite case. The case of finite cocktail party graphs was settled by Tosiro Tsuzuku in 1968 (Nagoya Math. J.). We give a separate, combinatorial proof that assumes $|\Omega| \ge 2,400$; this proof also works when Ω is infinite.

Keywords: permutation group; asymmetric coloring; sharply arc-transitive action

Gallai's path-decomposition conjecture

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Given a connected graph, how many disjoint paths do we need to cover all the edges? Gallai conjectured in 1968 that half the number of vertices (rounded up) is enough, which would be tight if true. We discuss recent progress and the obstacles to future works.

Keywords: graph decomposition; paths; Gallai's conjecture

Searching posets: from dichotomy to the golden ratio

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Searching a sorted array is a very classical textbook problem. In some special circumstances, such as searching a regression in a version control system, it is necessary to search in a partially ordered set. The problem can then be seen as trying to identify a faulty vertex in a directed acyclic graph where each step consists in querying a vertex and asking whether the searched vertex is an ancestor of the current vertex. Though the problem is known to be NP-complete in general, we put more focus on the more interesting setting when the indegree of each vertex is bounded, and in particular when it is bounded by two. We show that with the degree 2 restrictions, all directed acyclic graphs on n vertices can be searched in no more than $\log_\phi(n) + 1$ queries, where ϕ stands for the golden ratio. On the other hand, we also show that there exists directed trees which require at least $\lceil \log_\phi(n) \rceil - 2$ queries. Expectedly, the later proof expectedly involves Fibonnaci related trees. We will also discuss how the NP-completeness proof of the problem can be extended to this setting.

Joint work with Julien Courtiel and Romain Lecoq.

Keywords: regression search; posets; directed acyclic graphs; Fibonnaci sequence

Transversals in 6-uniform hypergraphs: A proof of the Tuza-Vestergaard Conjecture

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The transversal number $\tau(H)$ of a hypergraph H is the minimum number of vertices that intersect every edge of H. A 6-uniform hypergraph has all edges of size 6. In 2000 Tuza and Vestergaard conjectured that if H is a 3-regular 6-uniform hypergraph of order n, then $\tau(H) \leq \frac{1}{4}n$. We present a proof of this conjecture, which has become known as the Tuza-Vestergaard Conjecture. As an application, we present lower and upper bounds on the Tuza constant c_6 which is defined by

$$c_6 = \sup \frac{\tau(H)}{n(H) + m(H)},$$

where the supremum is taken over the class of all 6-uniform hypergraphs H. To determine the exact value of c_6 remains an open problem now for over 30 years. We show that $\frac{1}{6} \leq c_6 \leq \frac{1}{6} + \frac{1}{110}$, where the conjectured value is $c_6 = \frac{1}{6}$. Using an interplay with transversals in hypergraphs and total domination in graphs, we show that if G is a graph of order n with $\delta(G) \geq 6$, then $\gamma_t(G) \leq \frac{4549}{13299}n < \left(\frac{4}{13} + \frac{17}{494}\right)n$, where $\gamma_t(G)$ denotes the total domination number of G and $\gamma_t(G) \leq \frac{4}{13}n$ is conjectured to be the correct bound.

Joint work with Christian Löwenstein and Anders Yeo.

Keywords: transversals in hypergraphs; Tuza-Vestergaard conjecture; total domination

The imperial ruler

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The title of this talk can be interpreted in various ways. We will discuss these interpretations in connection with the life and work of Sandi Klavžar.

Keywords: distance in graphs; Sierpiński graph; Sierpiński triangle graph

Never trust a partial cube: an ode to the Desargues graph

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A graph is a partial cube if it is an isometric subgraph of a hypercube graph. Despite this seemingly restrictive definition, many important graphs, e.g., diagrams of distributive lattices, median graphs, linear extension graphs, Cayley graphs of Coxeter groups, dual graphs of hyperplane arrangements, fall into this class. As the title suggests, when studying partial cubes one might come up with false intuition suggested by their nice structure. A common bad guy is the 20-vertex Desargues graph. We will use this partial cube as a guide through several open problems and results in the area on topics such as planarity, Cayley graphs, and tope graphs of oriented matroids.

Keywords: partial cubes; planarity; Cayley graphs; tope graphs

Q-ary generalizations of set intersection and extremal graph theoretic problems

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Characteristic vectors of subsets of an n-element ground set give a natural 1-to-1 correspondence between set systems and 0-1 vector systems. As the size of the intersection of two sets equals the scalar product of their characteristic vectors, this correspondence is often used in proofs of intersection theorems of finite sets. There exist several definitions of intersection for vectors of length n with entries from $\{0, 1, ..., q\}$. In this talk, we will propose a new one: the size of the *s-sum intersection* of two such vectors u, v is the number of coordinates where the entries have sum at least s, i.e. $|\{i : u_i + v_i \geq s\}|$. We address analogs of the following classical results in this setting: the Erdős–Ko–Rado theorem and the theorem of Bollobás on intersecting set pairs. We will also define an s-sum analog of graph Turán problems and survey results concerning them.

Joint work with Zsolt Tuza and Máté Vizer.

Keywords: intersection theorems; graph Turán problems; extremal combinatorics

On well-dominated product graphs

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A graph X is called *well-dominated* if all of its minimal dominating sets have the same cardinality, namely $\gamma(X)$, the domination number of X. Several classes of well-dominated graphs have been characterized including bipartite graphs, chordal graphs, block graphs, unicyclic graphs, and graphs with girth at least 5.

In 2017 Gözüpek, Hujdurović, and Milanič gave a characterization of the well-dominated graphs that are nontrivial lexicographic products. In this talk I will present recent results concerning the class of well-dominated graphs that are nontrivial Cartesian, direct or strong products.

Joint work with Sarah Anderson and Kirsti Kuenzel.

Keywords: product graph; well-dominated; isolatable vertex

Distinguishing graph vertices by coloring the edges - palettes, automorphisms, local irregularity

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Let $G = (V, E)$ be a graph of order n. The function $f : E(G) \longmapsto \{1, 2, \ldots, k\}$ is called an *edge coloring*. Any such coloring assigns to the vertices of a graph *palette of colors*, *i.e.* the multiset of colors on incidental edges. Palettes are used to distinguish the vertices of the graph in one way or another.

A completely different approach to coloring distinguishing all vertices refers to the concept of automorphism of a graph. This approach is not related to local properties. It's about colorings that break down all non-trivial automorphisms.

Yet another approach is based on the concept of a locally irregular graph. A *locally irregular graph* is a graph in which every two adjacent vertices have distinct degrees. By a locally irregular decomposition of a graph, we thus mean a decomposition into locally irregular graphs.

The aim of the talk is discussion about the relationship between these different approaches to the problem of distinguishing vertices of a graph.

Keywords: edge coloring; automorphisms breaking a coloring; locally irregular graphs

Metric dimension in graphs: from distance vectors to distance multisets

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Given a simple and connected graph G, the *metric representation* of a vertex $u \in V(G)$ with respect to a set $S = \{w_1, \ldots, w_t\} \subseteq V(G)$, is the vector

$$r(u|S) = (d_G(u, w_1), \ldots, d_G(u, w_t)),$$

where $d_G(x, y)$ stands for the distance between the vertices x, y. The set $S \subset V(G)$ is a *resolving set* for G if $r(u|S) \neq r(v|S)$ for every $u, v \in V(G)$. The cardinality of the smallest such set is the *metric dimension* of G.

On the other hand, the *multiset representation* of $u \in V(G)$ with respect to S is the multiset

$$m(u|S) = \{\!\!\{d_G(u, w_1), \ldots, d_G(u, w_t)\}\!\!\}.$$

The set S is a *multsiset resolving set* if $m(u|S) \neq m(v|S)$ for every $u, v \in V(G) \setminus S$. The cardinality of the smallest such set is the *outer multiset dimension* of G.

Some short story on the evolution of the classical metric dimension into the outer multiset dimension of graphs shall be presented in this talk, by emphasizing on some recently developed contributions obtained in collaboration with Sandi Klavžar.

Joint work with Sandi Klavžar and Dorota Kuziak.

Keywords: metric dimension; outer multiset dimension; multiset resolving sets; resolving sets

List coloring of planar graphs

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The classical results of Thomassen and Voigt answered some fundamental questions about list coloring of planar graphs (i.e., every planar graph is 5-chooable, every planar graph of girth at least 5 is 3-choosable, and that there are planar graphs that are not 4-choosable, and there are planar graphs of girth 4 that are not 3-choosable). Yet there are still many interesting and challenging questions concerning this topic remain open. In this talk, I shall survey some recent progress on some of the problems. In particular, I shall sketch a proof of a recent result that every planar graph is $(4, 2)$-choosable, meaning that for any 4-list assignment L of a planar graph G, if $|L(x) \cap L(y)| \leq 2$ for each edge xy, then G is L-colorable. This answers a question asked by Kratochvíl, Tuza and Voigt in 1998. I shall also present some partial results on Škrekovski's conjecture that every planar graph is $(3, 1)$-choosable.

Joint work with Jialu Zhu, Rongxing Xu, and Yiting Jiang.

Keywords: graph coloring; planar graph; list with separation

Contributed talks

More on the graph pebbling

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Let $G=(V,E)$ be a simple graph. A function $\phi : V \to \mathbb{N}$ is called a configuration of pebbles on the vertices of G and the quantity $\sum_{u\in V}\phi(u)$ is called the size of ϕ which is just the total number of pebbles assigned to vertices. A pebbling step from a vertex u to one of its neighbors v reduces $\phi(u)$ by two and increases $\phi(v)$ by one. Given a specified target vertex r we say that ϕ is t-fold r-solvable if some sequence of pebbling steps places t pebbles on r. Conversely, if no such steps exist, then ϕ is r-unsolvable. The minimum positive integer m such that every configuration of size m on the vertices of G is t-fold r-solvable is denoted by $\pi_t(G,r)$. The t-fold pebbling number of G is defined to be $\pi_t(G) = max_{r\in V(G)}\pi_t(G,r)$. When $t = 1$, we simply write $\pi(G)$, which is the pebbling number of G. In this talk, after reviewing some previous results on $\pi(G)$, we state some new results on this parameter.

Joint work with F. Aghaei.

Keywords: pebbling; optimal pebbling; friendship graph

On Hosoya's dormants and sprouts

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The study of cospectral graphs is one of the traditional topics of spectral graph theory. Over the years many constructions of cospectral graphs were proposed, among which the Godsil-McKay local switching and the Schwenk's use of coalescences are most well-known, since both were were used to show (around the 1980s) that almost all trees have cospectral mates. Recently, enumerations of cospectral graphs with up to 12 vertices by Haemers and Spence and by Brouwer and Spence have led to the conjecture that, on the contrary, "almost all graphs are likely to be determined by their spectrum". This conjecture paved the way for myriad of results showing that various special types of graphs are determined by their spectra. On the other hand, in a recent series of papers, Hosoya drew the attention to a particular way of constructing cospectral graphs by using coalescences: that cospectral graphs can be constructed by attaching multiple copies of a rooted graph in different ways to subsets of vertices of an underlying graph.

After briefly surveying the history of constructing cospectral graphs, we focus on the expectations and questions about cospectrality of multiple coalescences that were raised in Hosoya's papers. In particular, we discuss the characteristic polynomial of such multiple coalescences, from which a necessary and sufficient condition for their cospectrality follows. We enumerate such pairs of cospectral multiple coalescences for a few families of underlying graphs, and show the infinitude of cospectral multiple coalescences having paths as underlying graphs, which were deemed rare by Hosoya.

Joint work with Ali Kanso and Dragan Stevanović.

Keywords: cospectral graphs; graph coalescence; removal-cospectral sets

Connected domination in random graphs

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Given a graph $G = (V, E)$, a *dominating set* is a subset $D \subseteq V$ such that every vertex in $V \setminus D$ is adjacent with at least one vertex in D. The *domination number* of G, denoted by $\gamma(G)$, is the minimum cardinality of a dominating set in G. Assuming that the graph $G = (V, E)$ is connected, a subset $D \subseteq V$ is said to be a *connected dominating set* if it is a dominating set and the subgraph $G[D]$ induced by $D \subseteq V$ is connected. The minimum cardinality of a connected dominating set is termed the *connected domination number*, denoted by $\gamma_c(G)$.

Comparing $\gamma(G)$ and $\gamma_c(G)$ for a random graph with constant edge probability p, we obtain that the two parameters are asymptotically equal with probability tending to 1 as the number of vertices gets large.

We also consider nonconstant edge probability p_n tending to zero (where n is the number of vertices). Among other results, we extend an asymptotic formula of Gilbert on the probability that a random graph is connected. Moreover, we present open problems in this new frame.

Joint work with József Túri and Zsolt Tuza.

Keywords: random graph; graph domination; connected domination

Domination in catacondensed hexagonal systems

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In this talk, we present some results about the domination number in catacondensed hexagonal systems, starting with the easiest ones, the hexagonal chains. A *hexagonal system* is a finite connected plane graph without cut vertices, in which all interior regions are mutually congruent regular hexagons. A hexagonal system is said to be *simple* if it can be embedded into the regular hexagonal lattice in the plane without overlapping of its vertices. Hexagonal systems that are not simple are called *jammed*. They are of great importance for theoretical chemistry because they are natural graph representations of benzenoid hydrocarbons. A *hexagonal chain* is a hexagonal system with the properties that (a) it has no vertex belonging to three hexagons, and (b) it has no hexagon with more than two adjacent hexagons (it is like a path of hexagons). A *catacondensed hexagonal system* is a hexagonal system which has no vertex belonging to three hexagons (it is like a tree of hexagons). We show some bounds on the domination number using the structure of the catacondensed hexagonal system, its number of hexagons and its number of branching hexagons.

Joint work with Robinson A. Higuita and Juan P. Rada.

Keywords: domination number; hexagonal system; catacondensed hexagonal systems

List covering of regular multigraphs

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A graph covering projection, also known as a locally bijective homomorphism, is a mapping between vertices and edges of two graphs which preserves incidencies and is a local bijection. This notion stems from topological graph theory, but has also found applications in combinatorics and theoretical computer science.

It has been known that for every fixed simple regular graph H of valency greater than 2, deciding if an input graph covers H is NP-complete. In recent years, topological graph theory has developed into heavily relying on multiple edges, loops, and semi-edges, but only partial results on the complexity of covering multigraphs with semi-edges are known so far. In this paper we consider the list version of the problem, called List-H-Cover, where the vertices and edges of the input graph come with lists of admissible targets. Our main result reads that the List-H-Cover problem is NP-complete for every regular multigraph H of valency greater than 2 which contains at least one semi-simple vertex (i.e., a vertex which is incident with no loops, with no multiple edges and with at most one semi-edge). Using this result we show the NP-co/polytime dichotomy for the computational complexity of List-H-Cover of cubic multigraphs.

Joint work with Jiří Fiala, Nikola Jedličková, Jan Kratochvíl, and Paweł Rzążewski.

Keywords: graph homomorphism; covering; multigraphs; semi-edges; complexity

Maturity models and knowledge spaces I - Modeling with partial cubes

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The longitudinal study aims to obtain information on changes in a cohort of subjects undergoing a common process being monitored. Additionally, it can be used to anticipate risks or predict future developments. The subjects mature in the studied process and observations of ordinal variables measuring the progress of subjects provide insight into the observed process.

Our primary interest in the subject arises from observing emotional states of students during a project-based learning course. Data from several one-semester project based courses provides insight into the link between motivation and development of skills. Moreover, it provides interesting graphs that allow us to apply graph theoretic tools to investigate these processes. One of the tools are (tile) crossing numbers whose minimization allows for ranking of the subjects at specific tests in the longitudinal study.

Another tool are partial cubes, which are used to model the learning space of the subjects; given this model of the learning space, the maturity model provides insight into a path through its graph, and the link between the difficulty of a path and the resources required to learn it can be optimized using our approach.

Joint work with Maša Galun, Janja Jerebic, and Špela Kajzer.

Keywords: longitudinal study; maturity model; tile crossing number; partial cube; optimal drawing

Even cycle decompositions of index 3 in 4-regular graphs

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An even cycle decomposition (ECD) of an Eulerian graph is a partition of the edge set into even cycles. It is known that every Eulerian graph containing no subgraph contractible to K_5 and containing no odd block has an ECD [4]. Nevertheless, the results in [4] do not solve the problem for 4-regular graphs since almost all 4-regular graphs have a K_5-minor [2].

We study ECDs in 4-regular graphs satisfying the following additional condition. We color the even cycles so as two cycles sharing at least one vertex get distinct colors. If k is the minimum number of required colors, then we say that the ECD has index k. We are interested in ECDs with the smallest index k. For a 4-regular graph G, there exists an ECD of index $k = 2$ if and only if G is class 1, otherwise $k \geq 3$ for every ECD. We prove the existence of an ECD of index 3 for some infinite families of 4-regular graphs. The results give a new contribution to the problem on the existence of ECDs in 4-regular graphs and to other open problems that are known in the literature, see [1, 3].

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Keywords: graph decompositions; Eulerian graphs; graph coloring

Grundy domination and zero forcing in regular graphs

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Given a finite graph G, the maximum length of a sequence $(v_1, \dots, v_k)$ of vertices in G such that each v_i dominates a vertex that is not dominated by any vertex in $\{v_1, \dots, v_{i-1}\}$ is called the Grundy domination number, $\gamma_{\rm gr}(G)$, of G. A small modification of the definition yields the Z-Grundy domination number, which is the dual invariant of the well-known zero forcing number. In this talk, we show that $\gamma_{\rm gr}(G) \geq \frac{n+\lceil \frac{k}{2} \rceil -2}{k-1}$ holds for every connected k-regular graph of order n different from K_{k+1} and $\overline{2C_4}$. The bound in the case $k = 3$ reduces to $\gamma_{\rm gr}(G) \geq \frac{n}{2}$, and we characterize the connected cubic graphs with $\gamma_{\rm gr}(G) = \frac{n}{2}$. If G is different from K_4 and $K_{3,3}$, then $\frac{n}{2}$ is also an upper bound for the zero forcing number of a connected cubic graph, and we characterize the connected cubic graphs attaining this bound.

Joint work with Boštjan Brešar.

Keywords: Grundy domination number; zero forcing; regular graph; cubic graph

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Domination game with shifts on caterpillars

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The domination game and the corresponding graph invariant $\gamma_g(G)$ were introduced by Brešar, Klavžar, and Rall in 2010. One of the central topics in the area is the so-called '3/5-conjecture.' As it was proved by Kinnersley, West, and Zamani in 2013, the conjecture is true for caterpillars i.e. $\gamma_g(G) \leq 3n/5$ holds for every caterpillar G of order $n \geq 2$. They also posed the question whether the upper bound can be improved for caterpillars of appropriately large orders. In the talk, we answer the question affirmatively by proving that every caterpillar of order n satisfies $\gamma_g(G) \leq \lceil 7n/12 \rceil$.

Keywords: domination game; caterpillar

On three domination-based identification problems in block graphs

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The problems of determining minimum identifying or (open) locating-dominating codes are special search problems that are challenging from both theoretical and computational viewpoints. In these problems, one selects a dominating set C from the vertex set $V(G)$ of a graph G such that the vertices of a chosen subset of $V(G)$ (e.g. $V(G) \setminus C$ or $V(G)$ itself) are uniquely determined by their neighborhoods in C. A typical line of attack for these problems is to determine tight bounds for the minimum codes in special graphs. In this work, we present for block graphs (i.e. diamond-free chordal graphs) tight lower and upper bounds for all the three codes, and examples of block graphs which attain these bounds. Our upper bounds are in terms of the number of maximal cliques $n_Q(G)$, the order $|V(G)|$ and other structural properties of a block graph G. As for the lower bounds, we prove them to be linear in terms of (i) $n_Q(G)$, and (ii) the order of G.

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Joint work with Florent Foucaud, Aline Parreau, and Annegret K. Wagler.

Keywords: identifying code; (open) locating-dominating; domination number; block graph

Distance measures in graphs of girth 6

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Let G be a connected graph. The diameter and Wiener index of G are defined as the largest and the sum of the distances between all pairs of vertices of G. The average eccentricity of G is the arithmetic mean of the eccentricities of the vertices of G, where the eccentricity of a vertex v is defined as the distance between v and a vertex farthest from v.

In this talk we present bounds on these distance measures for graphs of girth 6 in terms of order, minimum degree and maximum degree. We further show that, in a sense, our bounds are best possible. In this context a natural generalisation of cages arises, which is interesting in its own right. The cage problem considers the minimum order of regular graphs of prescribed degree and girth. In our generalisation we consider not regular graphs, but prescribe minimum and maximum degree.

Joint work with Alex Alochukwu.

Keywords: diameter; Wiener index; average eccentricity; girth; cage

Thresholds for monochromatic clique transversal game

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The (a, b)-monochromatic clique transversal game is a game played on a graph G by two players, Alice and Bob. The players alternately color vertices in G, in each move Alice colors a vertices red while Bob colors b vertices blue. We observe that this game is equivalent to the (b, a)-biased Maker-Breaker game played on the clique-hypergraph of G. The main results are estimating the thresholds bias $a_1(G)$ (res., $a'_1(G)$) which is the smallest integer a such that Alice can win in the Alice-start (res., Bob-start) $(a, 1)$-monochromatic clique transversal game on G. We determine $a_1(G)$ and $a'_1(G)$ where G is a triangle-free graph, establish lower and upper bounds on the disjoint union of graphs, and show that all values between the bounds are realizable. Moreover, we obtain the exact values of the thresholds for Cartesian products of paths and cycles.

Joint work with Csilla Bujtás and Sandi Klavžar.

Keywords: monochromatic clique transversal game; clique-hypergraph; biased Maker-Breaker game; triangle-free graph

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b-coloring of regular graphs

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We consider the problem of b-coloring of regular graphs. The b-chromatic number of a graph G is the largest integer k such that G admits a proper k-coloring in which every color class contains at least one vertex that has a neighbor in each of the other color classes. There is known the conjecture for this type of coloring of regular graphs posed in 2009.

Conjecture 1 *Every d-regular graph with girth at least 5, different from the Petersen graph, has a b-coloring with $d + 1$ colors.*

We present some new results towards proving the conjecture.

Keywords: b-coloring; regular graphs; graph coloring

Hamiltonicity and matchings in quartic graphs... the case of accordion graphs

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Let G be a graph of even order and let K_G be the complete graph on the same vertex set of G. A pairing of a graph G is a perfect matching of the graph K_G. A graph G has the Pairing-Hamiltonian property (the PH-property) if for each one of its pairings, there exists a perfect matching of G such that the union of the two gives rise to a Hamiltonian cycle of K_G. In 2015, Alahmadi *et al.* gave a complete characterisation of the cubic graphs having the PH-property, by showing that the only three cubic graphs having the PH-property are the complete graph K_4, the complete bipartite graph $K_{3,3}$, and the 3-dimensional cube. The next step would be to characterise the quartic graphs that have the PH-property.

In this talk we discuss a class of quartic graphs on two parameters, n and k, which we call the class of accordion graphs $A[n, k]$. We show that an infinite family of quartic graphs (which are also circulant) that Alahmadi *et al.* stated to have the PH-property are, in fact, members of this general class of accordion graphs. We study the PH-property of these accordion graphs, at times focusing on the pairings of G which are also perfect matchings of G. We investigate a close relationship that exists between accordion graphs and the Cartesian product of two cycles, and give a complete characterisation of those accordion graphs that are circulant graphs.

Joint work with Jean Paul Zerafa.

Keywords: Hamiltonian cycle; perfect matching; quartic graph; circulant graph

On the super domination number of a graph

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Let $G = (V, E)$ be a simple graph. A dominating set of G is a subset $S \subseteq V$ such that every vertex not in S is adjacent to at least one vertex in S. The cardinality of a smallest dominating set of G, denoted by $\gamma(G)$, is the domination number of G. A dominating set S is called a super dominating set of G, if for every vertex $u \in \overline{S} = V - S$, there exists $v \in S$ such that $N(v) \cap \overline{S} = \{u\}$. The cardinality of a smallest super dominating set of G, denoted by $\gamma_{sp}(G)$, is the super domination number of G. In this talk, I present some of recent results on the super domination number and a new definition in this field.

Keywords: domination number; super dominating set; unary and binary operations

Paired domination in trees

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Let G be a graph with no isolated vertices. We call a set of vertices $S \subset V(G)$ a *paired dominating* set if every vertex of the graph G is adjacent to a vertex in S and the subgraph induced by S contains a perfect matching. The *paired domination number* of the graph G, denoted $\gamma_{pr}(G)$, is the minimum number of vertices in a paired dominating set of G. This parameter has been defined and first studied by Haynes and Slater. Among other results, they proved that $\gamma_{pr}(G) \leq |V(G)| - 1$, if G is a connected graph of order at least three. They also characterized the graphs which achieve this upper bound. Except for two small cycles these graphs are the family of subdivided stars. This served as a motivation to consider the family of trees to possibly improve this general upper bound in terms of parameters other than the order of the graph.

In this talk, we present an upper bound on the paired domination number of a tree in terms of its order, maximum degree, and number of vertices of degree 1 and 2. We show that this bound is tight for all maximum degrees at least three. Furthermore, we present tight upper bound on the paired domination number of a tree in terms of its independence number.

Joint work with Michael A. Henning, Elżbieta Kleszcz, and Monika Pilśniak.

Keywords: paired domination; trees; independence number

On the cyclomatic number and (edge, mixed) metric dimension of connected graphs

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Let G be a finite, undirected and connected graph. Sedlar and Škrekovski recently conjectured that $\dim(G) \leq L(G) + 2c(G)$ and $\mathrm{edim}(G) \leq L(G) + 2c(G)$, where $\dim(G)$ and $\mathrm{edim}(G)$ are the metric and edge metric dimensions of G, $c(G)$ is the cyclomatic number of G, and $L(G)$ is the minimum number of vertices that are needed from so-called legs of the graph. Sedlar and Škrekovski also presented a similar conjecture for the mixed metric dimension of G; if $G \neq C_n$, then $\mathrm{mdim}(G) \leq l(G) + 2c(G)$, where $l(G)$ is the number of leaves in G. We show that $\dim(G)$ and $\mathrm{edim}(G)$ are bounded by $L(G) + 4c(G) - 4$ and that we have $\mathrm{mdim}(G) \leq l(G) + 4c(G) - 4$ when G is not a tree or a unicyclic graph.

Joint work with Dibyayan Chakraborty and Florent Foucaud.

Keywords: cyclomatic number; metric dimension; edge metric dimension; mixed metric dimension

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Efficient checking of diagram commutativity

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We are concerned with commutative diagrams in the usual sense: A diagram is a digraph for which the vertices are objects in a category and the arcs are morphisms A diagram commutes if the compositions along any two routes from one vertex to another are equal. The number of route pairs can increase exponentially with a diagram's size. Despite this, we describe an efficient means of deciding whether a diagram commutes.

Joint work with Paul Kainen.

Keywords: commutative diagrams; acyclic digraphs; bubbles in digraphs

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Relations of Wiener index and (revised) Szeged index on cacti

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The Wiener index is the sum of all distances in a graph and the (revised) Szeged index sums up contributions of edges. While not obviously related, they originate from the same concept on trees, and thus have close connections.

In this talk we discuss special relations of the Wiener index and the (revised) Szeged index on cactus graphs. Along the way we provide a representation of the revised Szeged index as sum over vertices.

Keywords: Wiener index; (revised) Szeged index; cactus graphs

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The game of cops and robber on geodesic spaces

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Recently, Mohar introduced a variation of the cops and robber game that is played on arbitrary geodesic spaces. The game combines properties of both the classical cops and robber game played on graphs and the differential pursuit-evasion games. In the game, cops win if they can get arbitrarily close to the robber. On the other hand, cops catch the robber if one of them occupies the same point as the robber. In this talk we will discuss some properties of the game, observe the difference between the number of cops needed to catch the robber and the number of cops needed to win the game, and investigate several strategies for the players – some of those arise from the game played on graphs, while others have not been studied on graphs yet.

Joint work with Bojan Mohar and Alexandra Wesolek.

Keywords: cops and robber game; geodesic space; strategies

Min orderings and list homomorphism dichotomies for signed graphs

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The CSP dichotomy conjecture has been recently established, but a number of other dichotomy questions remain open, including the dichotomy classification of list homomorphism problems for signed graphs. For a fixed signed graph $\widehat{H}$, the list homomorphism problem asks whether an input signed graph $\widehat{G}$ with lists $L(v) \subseteq V(\widehat{H}), v \in V(\widehat{G})$, admits a homomorphism f to $\widehat{H}$ with all $f(v) \in L(v), v \in V(\widehat{G})$. Usually, a dichotomy classification is easier to obtain for list homomorphisms than for homomorphisms, but for signed graphs it is not the case.

In this talk, I will show a structural classification in the special case of weakly balanced irreflexive and reflexive signed graphs (previously conjectured by Kim and Siggers). This generalizes previous results on weakly balanced signed trees, and weakly balanced separable signed graphs. The proof of irreflexive case depends on first deriving a theorem on extensions of min orderings of (unsigned) bipartite graphs, which is interesting on its own.

Joint work with Jan Bok, Richard Brewster, Pavol Hell, and Arash Rafiey.

Keywords: graph homomorphisms; computational complexity; signed graphs

Maturity models and knowledge spaces II - Maturity model crossing minimization

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A panel, or longitudinal, data set follows a given sample of individuals over time, and thus provides multiple observations on each individual in the sample. Based on such data, several maturity models can be developed to measure the ability of an individual for gradual improvement in a particular discipline. In this talk, observations for one ordinal variable over time will be studied and presented using tiled graphs with the provably smallest number of crossings.

Joint work with Drago Bokal, Maša Galun, and Špela Kajzer.

Keywords: longitudinal study; maturity model; crossing number; tile crossing number; optimal drawing

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Majority edge-colorings of graphs

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Motivated by some similar notions considered for vertex-colorings, we introduce the notion of majority edge-colorings of graphs: For a simple graph G, a coloring $c : E(G) \to [k]$ is called a majority k-edge coloring if, for every vertex u of G and every color α in $[k]$, at most half the edges incident with u have the color α. We prove the best result that every graph without pendant edges has a majority 4-coloring. We also address the question which graphs admit majority 3-edge colorings. Moreover, we investigate a natural variation of majority edge-colorings.

Joint work with Felix Bock, Johannes Pardey, Monika Pilśniak, Dieter Rautenbach, and Mariusz Woźniak.

Keywords: edge coloring; chromatic index; unfriendly partition

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On some metric properties of direct-co-direct product

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Direct-co-direct product $G \circledast H$ of graphs G and H is a graph on vertex set $V(G) \times V(H)$. Two vertices (g, h) and (g', h') are adjacent if $gg' \in E(G)$ and $hh' \in E(H)$ or $gg' \notin E(G)$ and $hh' \notin E(H)$. In this talk we present the distance formula for $G \circledast H$ in the case of connected non-complete factors G and H. We show that eccentricity of a vertex of $G \circledast H$ for connected non-complete graphs G and H is bounded by five. We characterize the disconnected cases of $G \circledast H$. We also present the distance formula in the case where at most one factor of $G \circledast H$ is connected. The distance between two vertices of $G \circledast H$ in such case is bounded by three, except in some small number of exceptions.

Joint work with Iztok Peterin.

Keywords: graph product; direct-co-direct product; distance; eccentricity

Coloring directed hypergraphs

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Inspired by earlier results about proper and polychromatic coloring of hypergraphs, we investigate such colorings of directed hypergraphs, that is, hypergraphs in which the vertex set of each hyperedge is partitioned into two parts, a tail and a head. We present a conjecture of D. Pálvölgyi and the author, which states that directed hypergraphs with a certain restriction on their pairwise intersections can be properly colored with two colors. Besides other contributions, our main result is a proof of this conjecture for 3-uniform directed hypergraphs. This result can be phrased equivalently such that if a 3-uniform directed hypergraph avoids a certain directed hypergraph with two hyperedges, then it admits a proper 2-coloring. Previously, only extremal problems regarding the maximum number of edges of directed hypergraphs that avoid a certain hyperedge were studied.

Keywords: proper coloring; polychromatic coloring; hypergraph; directed hypergraph

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General position sets and mutual visibility sets

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Let $G = (V(G), E(G))$ be a connected graph and $S \subseteq V(G)$. Then S is a *general position set* if no triple of vertices from S lie on a common geodesic in G. Two vertices from S are *mutually visible* if there is a shortest path between them without further vertices of S, and S is a *mutual-visibility set* if its vertices are pairwise mutually visible. General position sets and mutual-visibility sets were introduced and motivated by different applications not long ago. In this talk, some results on these two in some sense dual concepts will be presented.

Keywords: general position set; mutual visibility set; graph product

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Graphs with a unique maximum independent set up to automorphisms

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Let G be a graph and let $S \subset V(G)$ has a property $\mathcal{P}$. By $\rho(G)$ we denote maximum (respectively minimum) cardinality of a subset of $V(G)$ with property $\mathcal{P}$. We say that G is ρ-unique if there is exactly one set $S \subset V(G)$ of cardinality $\rho(G)$ with property $\mathcal{P}$. Graphs with unique maximum independent set have been studied, inter alia, by Hopkins and Staton, and later by Gunther, Hartnell and Rall.

A graph G is called ρ-iso-unique if for any two subsets $S_1, S_2 \subset V(G)$ of cardinality $\rho(G)$ there is an automorphism $\varphi \in \mathrm{Aut}(G)$, such that $\varphi(S_1) = S_2$. In our work, we start the investigation into α-iso-unique graphs by giving the characterization of such trees and partially generalizing the results on chordal graphs. We state some results about the problem complexity and some results concerning α-iso-unique cartesian products.

Joint work with B. Brešar, T. Dravec, and A. Gorzkowska.

Keywords: paired domination; trees; independence number

Domination sets in regular graphs

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Let G be a graph and $S \subseteq V(G)$. Then S is dominating set if every vertex v of G is at distance at most 1 from S. The size of minimum dominating set in G is its dominating number $\gamma(G)$. If S is dominating and independent, then it is independent dominating set. The size of minimum independent dominating set in G is its independent dominating number $i(G)$. Obviously, $\gamma(G) \leq i(G)$ and so $1 \leq \frac{i(g)}{\gamma(G)}$. Interesting is the upper bound. If G is k-regular graph and $3 \leq k \leq 6$ then it was shown by Babikir, Goddard, Henning, Lyle and Southey that $\frac{i(G)}{\gamma(G)} \leq \frac{k}{2}$. We extend this result to all $k \geq 3$. The proof is split into two parts according to the number of edges in the subgraph of G induced by S and we present it in the talk. The only extremal graphs, for which $\frac{i(G)}{\gamma(G)} = \frac{k}{2}$, are the complete bipartite graphs $K_{k,k}$.

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Joint work with Riste Škrekovski and Aleksandra Tepeh.

Keywords: dominating set; independent dominating set; regular graph

The language of self-avoiding walks

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Let X be an infinite, locally finite, deterministically edge labelled graph such that the label-preserving automorphism group of X acts with finitely many orbits on X. Important examples in this class are Cayley graphs of finitely generated groups, where each directed edge is labelled with its generator. The language of self-avoiding walks consists of all words which can be read along self-avoiding walks on X.
In this talk we discuss a recent characterisation of the language of self-avoiding walks on graphs without thick ends. This language is k-multiple context-free if and only if the size of all ends of X is at most $2k$. Moreover, our approach shows that the connective constant of any thin-ended quasi-transitive graph is an algebraic number.

Joint work with Florian Lehner and Wolfgang Woess.

Keywords: self-avoiding walks; infinite graphs; formal languages; graph decomposition

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Coloring vertices with neighborhood constraints

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We will survey several (proper) vertex colorings with a common property of having additional constraints given to the set of colors. Among the most representative ones is the 2-distance coloring, i.e., a proper coloring of the square of a graph, in which we require that in a neighborhood of any vertex, each color appears at most once. The most famous result regarding this topic might be that of Thomassen stating that any subcubic planar graph admits a 2-distance coloring with at most 7 colors. In recent years, many other colorings were studied and for almost all of them there are some surprising results and intriguing open problems, particularly in the class of planar graphs. We will take a look at them.

Keywords: vertex coloring; neighborhood constraint; conflict-free coloring

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Characterising 3-polytopes of radius one with unique realisation

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Let F be a planar, 3-connected graph of radius one on p vertices, with a vertices of degree three. We investigate the unigraphic degree sequences for such graphs. We get a clean answer e.g. for $a = 2$ and $a = 3$, and we also partially generalise.

Keywords: unigraphic degree sequence; planar graph; 3-polytopal graph; graph radius

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On compression schemes in hypercubes

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Given a subset of vertices $\mathcal{C}$ of a hypercube Q_n (equivalently a family of subsets of $[n]$), a sample compression scheme is, roughly speaking, a map that shortens the information about elements of $\mathcal{C}$ with the ability of reconstructing it. A long standing sample compression conjecture asks to linearly bound the size of the optimal sample compression schemes by the Vapnik-Chervonenkis (VC) dimension of an arbitrary $\mathcal{C}$.

In the talk we will present some recent results on proving the mentioned conjecture in the cases that $\mathcal{C}$ has nice geometric and combinatorial properties. In particular, we will focus on subsets $\mathcal{C}$ coming from hyperplane arrangements, oriented matroids, complexes of oriented matroids, etc., all being subfamilies of isometrically embeddable subgraphs of hypercubes, i.e. partial cubes.

Keywords: sample compression scheme; VC dimension; oriented matroids; partial cubes

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Paired and semipaired domination in near-triangulations

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A dominating set D of a graph G is paired dominating if the subgraph induced by D has a perfect matching and semipaired dominating if there is a partition of D into 2-sets such that the vertices of each 2-set are at distance at most 2. The paired domination number, denoted by $\gamma_{pr}(G)$, is the minimum cardinality of a paired dominating set of G and the semipaired domination number, denoted by $\gamma_{pr2}(G)$, is the minimum cardinality of a semipaired dominating set of G. A near-triangulation is a biconnected planar graph that admits a plane embedding such that all faces are triangles except possibly the outer face. In this talk, we show tight upper bounds on the paired and semipaired domination numbers for near-triangulations, concretely, $\gamma_{pr}(G) \leq 2\lfloor \frac{n}{4} \rfloor$ for any near-triangulation G of order n, $n \geq 4$, and $\gamma_{pr2}(G) \leq \lfloor \frac{2n}{5} \rfloor$ for any near-triangulation G of order n, $n \geq 5$, with some exceptions.

Joint work with Mercè Claverol, Carmen Hernando, Montserrat Maureso, and Javier Tejel.

Keywords: domination; near-triangulation; paired domination; semipaired domination

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Some topological indices and the golden ratio

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In the field of chemical graph theory, a topological index is a numerical parameter of a graph which characterize its topology and is usually graph invariant. Hosoya index, Wiener index and Zagreb index are three examples of well-known topological indices. In this talk, after reviewing some previous results on these indices, we obtain some relationships between these topological indices and the golden ratio.

Joint work with Saeid Alikhani.

Keywords: Zagreb index; Hosoya index; golden ratio; Fibonacci numbers

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Stern's diatomic sequence and two classes of 1-2 rooted trees

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Stern's diatomic sequence is defined by $s(0) = 0, s(1) = 1$, $s(2n) = s(n)$, $s(2n+1) = s(n) + s(n+1)$, $n \geq 1$. This sequence, like the more famous Fibonacci sequence, is ubiquitous and appears in many different problems. We derive a second-order recurrence relation for $s(n)$ and give its solution by means of continued fractions. Moreover, we discuss other known solutions which depend on the representation of n and give another solution in matrix form. Then we also show that $s(n)$ can be determined by the number of leaves of a subclass of 1-2 rooted trees of height h, known as *genealogical 1-2 rooted trees*. The *1-2 rooted trees* of height h are defined as those rooted trees of height h, where each vertex has at most two successors and all leaves are at the level h. The subclass of genealogical 1-2 rooted trees has the additional property that if a vertex has two successors, then one has only one successor and the other two successors. We give some properties of these graphs and determine their number. Finally, we show that the Fibonacci trees, that is the genealogical 1-2 rooted trees where the number of vertices at each level is a Fibonacci number, are the maximal genealogical 1-2 rooted trees of height h with respect to the number of vertices.

Keywords: Stern's diatomic sequence; continued fractions; genealogical 1-2 rooted trees; Fibonacci trees

On distance and strong metric dimension of modular product

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Modular product $G \diamond H$ of graphs G and H is a graph on vertex set $V(G) \times V(H)$. Two vertices (g, h) and (g', h') are adjacent if $g = g'$ and $hh' \in E(H)$, or $gg' \in E(G)$ and $h = h'$, or $gg' \in E(G)$ and $hh' \in E(H)$, or $gg' \notin E(G)$ and $hh' \notin E(H)$.

Let G be a graph. A u, v-geodesic P is *maximal* if P is not contained in any other geodesic different than P. Vertices of the strong resolving graph G_{SR} of G are all the end-vertices of all maximal geodesics in G. Moreover, $uv \in E(G_{SR})$ if there exists a maximal u, v-geodesic in G.

We present the distance formula for modular product. Later we describe all edges of a strong resolving graph of $G \diamond H$. This is used to demonstrate how to obtain strong metric dimension of a modular product on several infinite families of graphs.

Joint work with Cong X. Kang, Aleksander Kelenc, and Eunjeong Yi.

Keywords: modular product; distance; strong metric dimension; strong resolving graph

On emulational equivalence of combinatorial games and the game Hackenforb

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In combinatorial game theory, two impartial games are considered equivalent if their corresponding Grundy values are equal. Although the developed theory is an irreplaceable tool when questions of the winner and the optimal strategy are concerned, it does not tell too much about the structural (dis)similarity between games. In this presentation we shall talk about another notion of equivalence of impartial combinatorial games, the so-called “emulational equivalence." This concept is stronger than the Grundy equivalence, but weaker than the isomorphism between the game graphs. Intuitively, two games are emulationally equivalent if they reduce to the same game when positions that are “essentially the same" are not treated as distinct. We also define a new impartial combinatorial game on graphs called *Hackenforb* and show that various well-known combinatorial games (such as Nim, Subtraction game, Notakto, Chomp) are emulationally equivalent to an instance of Hackenforb. We hope that it is not too ambitious to believe that, in some future, Hackenforb can establish itself as a new common “language" for studying many different games, and that this could help in bringing out some new possible directions from which we can approach various open questions on combinatorial games and shed some light on them.

Joint work with Bojan Bašić and Nikola Milosavljević.

Keywords: combinatorial games; emulational equivalence; games on graphs; Hackenforb

On a Problem of Steinhaus

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Let N be a positive integer. A sequence $X = (x_1, x_2, \dots, x_N)$ of points in the unit interval $[0, 1)$ is *piercing* if $\{x_1, x_2, \dots, x_n\} \cap \left[\frac{i}{n}, \frac{i+1}{n}\right) \neq \emptyset$ holds for every $n = 1, 2, \dots, N$ and every $i = 0, 1, \dots, n-1$. In 1958 Steinhaus asked whether piercing sequences can be arbitrarily long. A negative answer was provided by Schinzel, who proved that any such sequence may have at most 74 elements. This was later improved to the best possible value of 17 by Warmus, and independently by Berlekamp and Graham.

Consider a more general variant of piercing sequences. Let $f(n) \geq n$ be an infinite nondecreasing sequence of positive integers. A sequence $X = (x_1, x_2, \dots, x_{f(N)})$ is *f-piercing* if $\{x_1, x_2, \dots, x_{f(n)}\} \cap \left[\frac{i}{n}, \frac{i+1}{n}\right) \neq \emptyset$ holds for every $n = 1, 2, \dots, N$ and every $i = 0, 1, \dots, n-1$. A special case of $f(n) = n + d$, with d a fixed nonnegative integer, was studied by Berlekamp and Graham. They noticed that for each $d \geq 0$, the maximum length of any $(n + d)$-piercing sequence is finite. Expressing this maximum length as $s(d) + d$, they obtained an exponential upper bound on the function $s(d)$, which was later improved to $s(d) = O(d^3)$ by Graham and Levy. Recently, Konyagin proved that $2d \leqslant s(d) < 200d$ holds for all sufficiently large d.

Using a different technique, exploiting e.g. the Farey fractions, we prove that the function $s(d)$ satisfies $\lfloor c_1 d \rfloor \leqslant s(d) \leqslant c_2 d + o(d)$ where $c_1 = \frac{\ln 2}{1-\ln 2} \approx 2.25$ and $c_2 = \frac{1+\ln 2}{1-\ln 2} \approx 5.52$, thus improving Konyagin's results. We also prove that there exists an infinite f-piercing sequence with $f(n) = \gamma n + o(n)$ if and only if $\gamma \geq \frac{1}{\ln 2} \approx 1.44$.

Joint work with Marcin Anholcer, Bartłomiej Bosek, Jarosław Grytczuk, Grzegorz Gutowski, Rafał Pyzik, and Mariusz Zając.

Keywords: piercing sequence; Farey fraction; problem of Steinhaus

Metric coordinates in graphs

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Given an ordered vertex subset $W = \{w_1, \ldots, w_n\}$ of a graph G, the metric coordinates of a vertex v with respect to W are $r(v|W) = (d(v, w_1), \ldots, d(v, w_n))$. The subset W is a resolving set of G if $r(u|W) = r(v|W)$ implies $u = v$, for every $u, v \in V(G)$. Thus, the set of n-vectors $\{r(v|W)\colon v \in V(G)\}$ is provided by the resolving set W.

In this talk, we address the reverse point of view of this relationship, that is, given $C \subseteq \mathbb{Z}^n$, are there a graph G and a resolving set W such that $C = \{r(v|W)\colon v \in V(G)\}$? If the answer is positive, are G and R unique? Moreover, we explore the role played by the strong product of paths in this problem.

Joint work with Mercè Mora.

Keywords: resolving set; metric coordinates; strong product

Computing and storing M-polynomials of (planar) graphs

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Let G be a graph and let $m_{i,j}(G)$, $i, j \geq 1$, be the number of edges uv of G such that $\{d_v(G), d_u(G)\} = \{i, j\}$. The M-polynomial of G is $M(G; x, y) = \sum_{i \leq j} m_{i,j}(G) x^i y^j$. A general method for calculating the M-polynomials for arbitrary graph families is developed. The method is further developed for the case where the vertices of a graph have degrees 2 and p, where $p \geq 3$, and further for such planar graphs. Database for storing M-polynomials will be presented. It features storing and querying M-polynomials over various parameters.

Joint work with Sandi Klavžar and Emeric Deutsch.

Keywords: M-polynomial; degree-based topological index; planar graph

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Special lattices of orthogonal projectors

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In this presentation three partial orders for complex matrices will be introduced; which are known in the literature as left star order, star order, and core order.

The main interest of these orders $\leq$ is maybe the fact that, given any complex square matrix B of size n and rank r, it is possible to define an order-isomorphism between the corresponding interval $[O, B]_{\leq}$ and a certain poset of orthogonal projectors of size r.

In addition, all the involved posets can be proved to be lattices. Such lattice structure will be analyzed, and the eventual existence of infimums and supremums will be characterized.

Joint work with C. R. Cimadamore, L. A. Rueda, and N. J. Thome.

Keywords: Hartwig-Spindelböck factorization; left star partial order; star partial order; core partial order; lattice structure

Approximating energy of wine glass cycles and paths

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The energy $E(G)$ of a graph G is the sum of absolute values of the eigenvalues of its adjacency matrix. Recently, Akbari, Alazemi and Andjelić [Applicable Analysis and Discrete Mathematics 15 (2021), 444–459] proved that $E(G) \leq 2\mu(G)\Delta(G)$ holds when G is connected and $\Delta(G) \geq 6$, where $\mu(G)$ is the matching number and $\Delta(G)$ is the maximum vertex degree of G. Despite relying extensively on the bound $\Delta(G) \geq 6$ in the proof, they conjectured that the same inequality is also valid when $2 \leq \Delta(G) \leq 5$. Here we first showcase the process of finding small counterexamples to this conjecture by software tools. These small counterexamples lead to two aptly named infinite families—wine glass cycles and paths. We illustrate an elegant use of elementary calculus to approximate energy of these graphs.

Joint work with Ivan Damnjanović and Djordje Stevanović.

Keywords: graph energy; matching number; counterexamples

The odd coloring

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A proper vertex coloring φ of graph G is said to be odd if for each non-isolated vertex $x \in V(G)$ there exists a color c such that $\varphi^{-1}(c) \cap N(x)$ is odd-sized. The minimum number of colors in any odd coloring of G, denoted $\chi_o(G)$, is the odd chromatic number. Odd colorings were recently introduced in [M. Petruševski, R. Škrekovski: *Colorings with neighborhood parity condition*]. In the talk we discuss various basic properties of this new graph parameter, establish several upper bounds, several charatcterizatons, and pose some questions and problems. We will also consider another new and related coloring, so called the proper conflict-free coloring.

Joint work with Mirko Petruševski and Yair Caro.

Keywords: graph coloring; planar graph; four color theorem

The competition-independence game with prevention

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The competition-independence game is played on a simple finite graph G by two players, Diminisher and Sweller. The players alternate turns in choosing a vertex that is not adjacent to any of the vertices already chosen. The goal of Sweller is to make the final set of chosen vertices as big as possible while Diminisher's goal is just the opposite. If both players play optimally according to their goals two graph invariants, called the competition-independence numbers, are obtained. The first refers to a game started by Diminisher and the second to a game started by Sweller.

In this talk, we introduce a variation of the competition-independence game in which on each turn a player can decide to make any of the two types of moves, which are choosing a vertex (the chosen vertex has been added into the independent set that is being built during the game) and preventing a vertex from being chosen (the prevented vertex should not be played until the rest of the game). Given a graph G, $\widetilde{I}_{\rm d}(G)$ (resp. $\widetilde{I}_{\rm s}(G)$) denotes the size of the final independent set of chosen vertices in the competition-independence game with prevention in which both players play optimally and Diminisher (resp. Sweller) moves first.

We use the Partition Strategy to determine the competition-independence numbers with prevention of paths. Further, we establish the general bounds, $1 \leq \widetilde{I}_{\rm d}(G) \leq \lfloor \frac{n}{2} \rfloor$ and $1 \leq \widetilde{I}_{\rm s}(G) \leq \lceil \frac{n}{2} \rceil$, and give the characterization of the classes of (connected) graphs G that attain the given bounds. Additionally, for graphs with diameter 2, we establish a close connection between the competition-independence games with prevention and a version of the coloring game called the packing coloring game.

Joint work with Boštjan Brešar.

Keywords: competition-independence game; independent domination game; coloring game; packing coloring

Injective coloring of planar graphs with maximum degree 4

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An *injective coloring* of a graph G is a coloring of the vertices of G in which every pair of vertices with a common neighbor receive distinct colors. The *injective chromatic number*, denoted by $\chi^i(G)$, is the smallest integer k such that there exists an injective coloring of G with k colors. The injective coloring was first introduced in 2002 by Hahn, Kratochvíl, Širáň, and Sotteau. This concept is also related to 2-distance coloring and exact square coloring. However, unlike the 2-distance coloring, both the injective coloring and the exact square coloring need not be proper.

In this talk we present an overview of the results related to the injective coloring of planar graphs. In particular, we focus on injective coloring of planar graphs with maximum degree $\Delta = 4$. We prove that planar graphs with $\Delta = 4$ are injectively list $(\Delta + 7)$-colorable. In addition, we prove that planar graphs with $\Delta = 4$ and girth at least 4 are injectively list $(\Delta + 5)$ colorable.

Joint work with Hoang La.

Keywords: injective coloring; distance coloring; planar graphs

Keeping the safety distance

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We introduce the novel notion of the span of graphs. Using this, we solve the problem of determining the *maximal safety distance* two players can keep at all times while traversing a graph (visit either all vertices, or all edges) with accordance to certain move rules. For this purpose, we introduce different variants of a span of a given connected graph. For each variant, we show that the solution can be obtained by considering only connected subgraphs of a graph product and the projections to the factors. We characterise graphs in which it is impossible to keep a positive safety distance at all moments in time. Wa also show that the chosen span variant of a given graph can be determined in polynomial time.

Joint work with Iztok Banič.

Keywords: strong span of a graph; direct span of a graph; Cartesian span of a graph; safety distance

Decompositions of uniform hypergraphs into tight cycles

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An r-uniform (tight) k-cycle is a cyclic sequence v_1, v_2, $\ldots$, v_k of k vertices, together with k edges which are formed by the r-tuples of consecutive vertices $v_i, v_{i+1}, \ldots, v_{i+r-1}$ ($i = 1, 2, \ldots, k$, subscript addition taken modulo k). A k-cycle decomposition of an r-uniform hypergraph $\mathcal{H}$ is a collection of r-uniform k-cycles that partition the edge set of $\mathcal{H}$. Arithmetic necessary conditions can be given for decomposability. We present new classes of hypergraphs for which those standard necessary conditions are also sufficient. For instance, we solve the problem for complete 3-uniform hypergraphs of order n minus $n/3$ disjoint edges in case of 9-cycles.

Joint work with Anita Keszler.

Keywords: hypergraph; edge decomposition; tight cycle; hypercycle system; block design

Resonant graphs of catacondensed coronoids

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A catacondensed coronoid H is a connected subgraph of a hexagonal system such that any edge of H lies in a hexagon of H, any triple of hexagons of H has empty intersection and the inner dual of H is a cactus graph. A perfect matching M of a catacondensed coronoid H is resonant if every cycle of the inner dual of H admits a hexagon h such that M possesses three edges of h. The vertex set of the resonant graph of a catacondensed coronoid H consists of all resonant perfect matchings of H, two perfect matchings being adjacent whenever their symmetric difference forms the edge set of a hexagon of H. A labeling which assigns a binary string to every resonant perfect matching of a catacondensed coronoid is presented. Given a coronoid H with n hexagons, the introduced labeling defines an isometric embedding of the resonant graph of H into the n-cube. Moreover, it is proved that the resonant graph of H is median. The introduced labeling allows us to show that the resonant graph of a class of graphs that belongs to catacondensed coronoid is isomorphic to a Lucas cube.

Keywords: catacondensed coronoid; perfect matching; resonant graph

Constructing sparsest ℓ-hamiltonian saturated k-uniform hypergraphs

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Given $k \geq 3$ and $1 \leq \ell < k$, an (ℓ, k)-cycle is one in which consecutive edges, each of size k, overlap in exactly ℓ vertices. We study the smallest number of edges in k-uniform n-vertex hypergraphs which do not contain hamiltonian (ℓ, k)-cycles, but once a new edge is added, such a cycle is promptly created. It has been conjectured [1, 2] that this number is of order n^{ℓ} and confirmed [2, 3] for $\ell \in \{1, k/2, k-1\}$, as well as for the upper range $0.8k \leq \ell \leq k-1$. Here we extend the validity of this conjecture to the lower-middle range $(k-1)/3 \leq \ell < k/2$.

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Joint work with Andrzej Ruciński.

Keywords: saturation; hamiltonian; cycle; hypergraph

Molecular descriptors of heteratomic and multiple bonds molecules

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Hydrocarbons are in general modelled with simple graphs but in the case of unsaturated hydrocarbons with multiple bonds there is some ambiguity in modelling such graphs, and consequently in the calculation of topological indices or molecular descriptors. We introduce different models for representing such molecules with an edge-weighted graphs and calculate the Wiener index. The regression analysis on the obtained weighted Wiener indices is performed regarding the prediction of boiling points of alkenes and alkadienes.

Vertex-weighted graphs are used to model corrosion inhibitors which are heteroatomic molecules. Various topological indices of these molecules are calculated and compared with theirs experimentally measured corrosion inhibition effectiveness. In the regression analysis the best predictors of corrosion inhibition effectiveness are obtained.

Joint work with Simon Brezovnik, Matjaž Finšgar, Slavko Radenković, Izudin Redžepović, and Niko Tratnik.

Keywords: molecular descriptor; topological index; Wiener index; unsaturated hydrocarbon; corrosion inhibition effectiveness