Complex Analysis, Geometry, and Dynamics 2022

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Embedding open Riemann surfaces into $\mathbb{R}^4$ by harmonic functions

Antonio Alarcón

Universidad de Granada

We shall prove that every open Riemann surface admits a proper harmonic embedding into $\mathbb{R}^4$. This reduces by one the previously known embedding dimension in this framework, due to Greene and Wu and dating back to 1975.

This is a joint work with Francisco J. López.

The Hermitian geometry of the Chern connection

Daniele Angella

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We consider some analytic problems concerning the geometry of the Chern connection of Hermitian manifolds, e.g.: the existence of metrics with constant Chern-scalar curvature; the generalizations of the Kähler-Einstein condition to the non-Kähler setting; the convergence of the normalized Chern-Ricci flow on compact complex surfaces.

This is joint work with Simone Calamai, Francesco Pediconi, Cristiano Spotti, Valentino Tosatti

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Tame sets in homogeneous spaces

Rafael Andrist

American University of Beirut

The notion of a tame set has been introduced by Rosay and Rudin in 1988 for closed discrete sets in $\mathbb{C}^n, n \geq 2$. Recently, the notion of a tame set has been generalized to complex manifolds. We prove the existence of tame sets in affine algebraic homogenenous spaces of complex linear algebraic Lie groups. Joint work with R. Ugolini.

Rank One limit functions

Anna Miriam Benini
Università di Parma

A Fatou component U for an automorphism F of $\mathbb{C}^2$ is a maximal open set on which the family of iterates $(F^n)_{n\in\mathbb{N}}$ is normal, that is, every sequence has a convergent subsubsequence. Any holomorphic function h that arises as such limit is called a *limit function*. Roughly speaking, an invariant Fatou component is called *recurrent* if $h(U) \subset U$ and *nonrecurrent* otherwise - since U is invariant, in this case, $h(U)$ is contained in the boundary of U. For several classes of functions, including polynomial automorphisms, there is a good understanding of recurrent Fatou components. On the other hand, major questions remain for nonrecurrent ones. One of the obstacles to a classification of nonrecurrent component is the establishment of conditions under which a limit function can have rank 1 (in other words, $h(U)$ has generic complex dimension 1). We will present some overview of the examples that we know of Fatou components with rank-1 limit functions in $\mathbb{C}^2$ (Jupiter-Lilov; BocThaler-Bracci-Peters) and in the line at infinity (B-Saracco-Zedda); and some cases in which it is possible to rule out the existence of such rank-1 limit functions (Lyubich-Peters).

Abstract boundaries and applications

Filippo Bracci
Università di Roma "Tor Vergata"

In this talk I will survey on some abstract boundary and related constructions which can be constructed using invariant metrics on complex manifolds (mainly our focus will be on domains) and applications such as extension of biholomorphisms and Denjoy-Wolff type theorem.

Quasiconformal folding: trees, triangles and tracts

Christopher Bishop
Stony Brook University

Quasiconformal folding is a type of quasiconformal surgery that carefully introduces critical points in order to construct holomorphic functions of one variable, with good control of both the geometry and the singular values. The method will be illustrated by a discussion of true trees (i.e., a special case of Grothendieck's dessins d'enfants on the sphere), equilateral triangulations of Riemann surfaces, a strengthening of Runge's theorem, and the construction of entire functions with specified geometry and bounded singular sets (the Eremenko-Lyubich class).

Flexible versus hyperbolic domains in $\mathbb{R}^n$

Barbara Drinovec Drnovšek
University of Ljubljana, and Institute of Mathematics, Physics and Mechanics

We shall focus on how the geometry of a domain $\Omega \subset \mathbb{R}^n$ influences the conformal properties of minimal surfaces that it contains. We define a Finsler pseudometric, g_Ω, like the Kobayashi pseudometric but using conformal minimal discs. The domain Ω is *hyperbolic* if g_Ω induces a distance function. In this case, Ω does not admit any conformal minimal surfaces parameterized by $\mathbb{C}$. In the opposite direction, we call the domain Ω *flexible* if every conformal minimal immersion $U \to \Omega$ from a Runge domain U in an open conformal surface M can be approximated uniformly on compacts by conformal minimal immersion $M \to \Omega$. This is joint work with Franc Forstnerič.

Kähler geometry and obstruction flat CR manifolds

Peter Ebenfelt
University of California

Let $X = X^{2n+1}$ be a C^∞-smooth strictly pseudoconvex CR hypersurface in a complex manifold of dimension $n+1$. There exists a local defining function ρ with a *finite* degree of smoothness, namely $C^{n+3-\epsilon}$, up to X such that $\omega = -i\partial\bar{\partial}\log\rho$ defines a Kähler–Einstein metric on the pseudoconvex side of X. In general, higher degree smoothness of ρ up to X is obstructed by a log-term whose coefficient is a local CR invariant of X. The CR manifold X is said to be *obstruction flat* if this obstruction invariant vanishes. It is easy to see that if X is spherical, then it is obstruction flat. For $n = 1$, the conjecture is, and there is ample evidence that the converse holds, i.e., if $X = X^3$ is compact and obstruction flat, then it is spherical. For $n \geq 2$, this is no longer true and the classification of compact, obstruction flat CR manifolds of higher dimension is wide open. In this talk, we shall discuss the special case where the CR manifold X arises as the unit circle bundle of a negative Hermitian line bundle over a Kähler manifold.

The monomial basis projection onto A^p spaces on Reinhardt domains

Luke Edholm

University of Vienna

This talk will address fundamental questions about A^p Bergman spaces in Reinhardt domains. The classical Bergman projection and its kernel function are well studied objects in the theory of several complex variables. On a large class of smoothly bounded pseudoconvex domains, the Bergman projection can be seen to extend from an orthogonal projection from L^2 onto A^2 to an absolutely bounded operator from L^p onto A^p, $1 < p < \infty$. But more recently, examples of non-smooth (but still pseudoconvex) domains with irregular Bergman have been given. In particular, we highlight the following deficiencies:

1. The Bergman projection may fail to be a bounded operator on L^p for $p \neq 2$.
2. Even if bounded on L^p, the Bergman "projection" may fail to be a surjective operator onto the full A^p for certain $p < 2$. (So calling it a projection is an abuse of language here.)
3. The Bergman projection may in fact fail to even be a closable operator on L^p for certain p.

After illustrating these failures I will introduce a new integral operator called the monomial basis projection, which can be defined on any pseudoconvex Reinhardt domain and any $1 < p < \infty$. I will then prove many properties about this operator and its representative integral kernel. Finally, I will demonstrate on a large class of domains that this operator beautifully sidesteps the problems above, and is thus a more suitable operator to study A^p spaces than the Bergman projection.

This is joint work with Debraj Chakrabarti.

Slice Conformality and Riemann Manifolds on Quaternions and Octonions

Graziano Gentili

Dipartimento di Matematica e Informatica "U. Dini", Università di Firenze, Italy

Quaternionic and octonionic analogs of the classical Riemann surfaces will be presented. The construction of these manifolds has some peculiarities, and the scrutiny of the classical approach to Riemann surfaces, mainly based on conformality, leads to the definition of slice conformal or slice isothermal parameterization of quaternionic or octonionic Riemann manifolds. These classes of manifolds include slice regular quaternionic and octonionic curves, graphs of slice regular functions, the 4 and 8 dimensional spheres, the helicoidal and catenoidal 4 and 8 dimensional manifolds. Appropriate Riemann manifolds help to give a unified definition of the quaternionic and octonionic logarithm and n-th root function. This is joint work with Jasna Prezelj and Fabio Vlacci.

On neighborhoods of embedded complex tori

Xianghong Gong
University of Wisconsin

In this talk, we will show that an n-dimensional complex torus embedded in a complex manifold has a neighborhood that is biholomorphic to a neighborhood of the zero section in the normal bundle of the torus, provided that the restriction of the tangent bundle of the complex manifold to the torus splits as the direct sum of the tangent and normal bundle of the torus, the normal bundle of the torus has (locally constant) Hermitian transition functions, and the normal bundle satisfies a non-resonant Diophantine condition. This is joint work with Laurent Stolovitch.

Holomorphic extension of multi-valued holomorphic functions on a Stein space with isolated singularities

Xiaojun Huang
Rutgers University

Classical Hartogs theorem says a single-valued holomorphic function extends to a whole domain if it is holomorphic at most away from a compact subset. The multi-valued Hartogs phenomenon was first studied by Kerner in 1962 for Stein manifold. A version of the classical Kerner's theorem for a singular Stein space with a compact strongly pseudoconvexboundary has been recently established by Huang-Xiao (Crelle's 2021) when the space has complex dimension at least 3. In this talk, I will discuss a joint paper with X. Li that answers the two dimensional case left open in the work of Huang-Xiao.

Factorization of holomorphic matrices

Frank Kutzschebauch
University of Berne

Every complex symplectic matrix in $\mathrm{Sp}_{2n}(\mathbb{C})$ can be factorized as a product of the following types of unipotent matrices (in interchanging order).

- (i): $\begin{pmatrix} I & B \\ 0 & I \end{pmatrix}$, upper triangular with symmetric $B = B^T$.
- (ii): $\begin{pmatrix} I & 0 \\ C & I \end{pmatrix}$, lower triangular with symmetric $C = C^T$.

The optimal number $T(\mathbb{C})$ of such factors that any matrix in $\mathrm{Sp}_{2n}(\mathbb{C})$ can be factored into a product of T factors has recently been established to be 5 by Jin, P. Lin, Z. and Xiao, B.

If the matrices depend continuously or holomorphically on a parameter, equivalently their entries are continuous functions on a topological space or holomorphic functions on a Stein space X, it is by no means clear that such a factorization by continuous/holomorphic unipotent matrices exists. A necessary condition for the existence is the map $X \to \mathrm{Sp}_{2n}(\mathbb{C})$ to be null-homotopic. This problem of existence of a factorization is known as the symplectic Vaserstein problem or Gromov-Vaserstein problem. In this talk we report on the results of the speaker and his collaborators B. Ivarsson, E. Low and of his Ph.D. student J. Schott on the complete solution of this problem, establishing uniform bounds $T(d,n)$ for the number of factors depending on the dimension of the space d and the size n of the matrices. It seems difficult to establish the optimal bounds. However we obtain results for the numbers $T(1,n)$, $T(2,n)$ for all sizes of matrices in joint work with our Ph.D. students G. Huang and J. Schott. Finally we give an application to the problem of writing holomorphic symplectic matrices as product of exponentials.

Levi flat hypersurfaces, singular webs, and Fuchsian systems on the Riemann sphere

Ilya Kossovskiy

Masaryk University in Brno & TU Wien

We discuss the classification of nondicritical singularities of Levi flat hypersurfaces in the complex 2-space. According to a structural result due to Shafikov-Sukhov, one can approach to the problem through the classification of implicit differential equations or alternatively singular 2-webs. The latter classification is supposed to be completed in the classical work of Davydov. We revisit the Davydov theory, by correcting the classification and extending it to the analytic setting. Interestingly, the problem appears to be closely connected to the seemingly unrelated problem of studying solutions for a Fuchsian system of ODEs on the Riemann sphere.

The Levi-core of a pseudoconvex domain

Samuele Mongodi

Università degli Studi di Milano-Bicocca

In a recent paper, with G.M. Dall'Ara, we defined the Levi core of a pseudoconvex CR manifold of hypersurface type; in this talk I would like to introduce this CR invariant, explain its link with the Diederich-Fornaess index and discuss its geometric properties, in particular its link with the Jensen boundary, its relations with other known properties such as being weakly regular or satisfying Catlin's property (P). I will also discuss in some detail the core of real analytic domains and how the procedure of reduction to the core mirrors Kohn's algorithm of subelliptic multiplier ideals.

Chaos and phase transitions on regular lattices

Han Peters
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Partition functions on graphs that are in some sense recursively connected often naturally leads to holomorphic dynamical systems. An elegant example is giving in the recent work of Ombra and Riveira-Letelier, where they study the partition function of the hard core model on Cayley trees. This setting induces the iteration of a one-parameter family of rational functions. In fact, the zero locus of the partition functions coincides with the bifurcation locus of the associated family of rational maps. As a consequence it was shown that there exists a single phase transition of infinite order.

In recent works with de Boer, Buys, Guerini and Regts, we demonstrated that this connection between the zero locus of partition functions and the bifurcation locus of associated rational functions persists in settings that have no clear dynamical interpretation. Examples are the family of all bounded degree graphs, for both the hard-core and the Ising model. Again the zero locus coincides with the interpretation of the bifurcation locus.

In current work we focus on the setting that is most interesting from a physical perspective: graphs converging to a regular lattice. While there is no clear interpretation as a holomorphic dynamical systems, both simulations and preliminary results demonstrate the potential of methods from complex dynamical systems in this setting.

Symplectic geometry, almost-complex structures, and the Type IIA flow

Duong H. Phong
Columbia University

The Type IIA flow is a flow of closed and primitive 3-forms introduced by the speaker in joint work with T. Fei, S. Picard, and X.W. Zhang on any 6-dimensional compact symplectic manifold. More than 20 years ago, Hitchin had shown how a 3-form on a 6d manifold defines an almost-complex structure. We find that on a symplectic manifold, this almost-complex structure becomes very rich if the 3-form is closed and primitive. The resulting manifold has SU(3) holonomy with respect to the projected Levi-Civita connection, the Nijenhuis tensor has only 6 independent components, and the Type IIA flow becomes a perturbation of the Kähler-Ricci flow which is expected to produce the optimal almost-complex structure compatible with the given symplectic structure.

On compacts possessing strictly plurisubharmonic functions

Nikolay Shcherbina
University of Wuppertal

We give a geometric condition on a compact subset of a complex manifold which is necessary and sufficient for the existence of a smooth strictly plurisubharmonic function defined in a neighbourhood of this set.

Polynomials with exponents in compact convex sets and associated weighted extremal functions

Ragnar Sigurdsson
University of Iceland

The Runge-Oka-Weil theorem states that if K is a compact polynomially convex subset of $\mathbb{C}^n$, i.e.,

$$K = \widehat{K} = \{z \in \mathbb{C}^n \,;\, |p(z)| \leq \|p\|_k, \ \forall p \in \mathcal{P}(\mathbb{C}^n)\}$$

and f is a holomorphic function in some neighborhood of K, then f can be approximated uniformly on K by polynomials.

If we let $\mathcal{P}_m(\mathbb{C}^n)$ denote the space of polynomials of degree $\leq m$ in n complex variables and let

$$d_{K,m}(f) = \inf\{\|f - p\|_K \,;\, p \in \mathcal{P}_m(\mathbb{C}^n)\}$$

denote the smallest error in an approximation of f by polynomials of degree $\leq m$, then the conclusion of Runge-Oka-Weil theorem is simply

$$\lim_{m\to\infty} d_{K,m}(f) = 0.$$

In 1962 Siciak intruduced his plurisubharmonic extremal functions

$$\Phi_K = \overline{\lim_{m\to\infty}}\, \Phi_{K,m}, \qquad \Phi_{K,m} = \sup\{|p|^{1/m} \,;\, p \in \mathcal{P}_m(\mathbb{C}^n), \|p\|_K \leq 1\}.$$

They appear in the Bernstein-Walsh-Siciak theorem: Assume that Φ_K is continuous. Then a function f satisfying the conditions in the Runge-Oka-Weil theorem has a holomorphic extension to the open set

$$\Omega_R = \{z \in \mathbb{C}^n \,;\, \Phi_K(z) < R\}$$

if and only if

$$\overline{\lim_{m\to\infty}} \big(d_{K,m}(f)\big)^{1/m} = \frac{1}{R}.$$

The lecture is a report on a joint work with Benedikt Steinar Magnússon, Álfheiður Edda Sigurðardóttir, and Bergur Snorrason, where we introduce a graded subring $\mathcal{P}^S(\mathbb{C}^n) = \cup_{m\in\mathbb{N}} \mathcal{P}^S_m(\mathbb{C}^n)$ of the usual polynomial space $\mathcal{P}(\mathbb{C}^n)$ and give a version of the Bernstein-Walsh-Siciak with weighted supremum norms on K with respect to weights of the form e^{-mq}.

Density Property for Vector Bundles

Jörg Winkelmann

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A complex manifold is said to enjoy the *density property* (in the sense of Varolin) if the complete vector fields generate a dense Lie subalgebra of the Lie algebra of all vector fields.

Let X be a Stein manifold and let $E \to X$ be a vector bundle. We show: If X enjoys the density property, so does E. In the algebraic context a similar result concerning flexibility is provided.

This is joint work with Riccardo Ugolini.

An extended Monge-Ampère operator

Elizabeth Wulcan

Chalmers University of Technology

I will discuss a joint work with Mats Andersson and David Witt Nyström, that extends previous work with Andersson and Błocki. We introduce a class of plurisubharmonic functions $\mathcal{G}$, for which there is a natural Monge-Ampère operator with nice local and global properties. The class $\mathcal{G}$ includes plurisubharmonic functions with analytic singularities and has certain convexity properties, and thus it has a quite rich structure.
