Lattice Homomorphisms and Orthogonally additive polynomials on Riesz spaces

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Positivity XI
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- A multilinear mapping $T : E^n \longrightarrow F$ is said to be orthosymmetric if $T(x_1, ..., x_n) = 0$ whenever $x_1, ..., x_n \in E$ satisfy $x_i \perp x_j$ for some $i \neq j$.
- Let E be a vector lattice and let F be a topological space. A map $P : E \to F$ is called a homogeneous polynomial of degree n (or a n-homogeneous polynomial) if $P(x) = \psi(x, .., x)$, where ψ is a n-multilinear map from E^n into F.
- A homogeneous polynomial, of degree n, $P : E \to F$ is said to be orthogonally additive if $P(x + y) = P(x) + P(y)$ where $x, y \in E$ are orthogonally (i.e. $|x| \wedge |y| = 0$).
- We denote by $\mathcal{P}_0(^nE, F)$ the set of n-homogeneous orthogonally additive polynomials from E to F.

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Introduction

One of the relevent problems in Operator Theory is to describe orthogonally additive polynomials via linear operators. This problem can be treated in different manner, depending on domains and codomains on which polynomials act. Interest in orthogonally additive polynomials on Banach lattices originates in the work of **Sundaresan**, where the space of n-homogeneous orthogonally additive polynomials on the Banach lattices l_p and $L_p[0,1]$ was characterized. It is only recently that the class of such mappings have been getting more attention. We are thinking here about works on orthogonally additive polynomials and holomorphic functions and orthosymmetric multilinear mappings on different Banach lattices and also $\mathbb{C}^*$-algebras. Proofs of the aforementioned results are strongly based on the representation of this spaces as vector spaces of extended continuous functions. So they are not applicable to general Riesz spaces. That is why we need to develop new approaches. Actually, the innovation of this work consist in making a relationship between orthogonally additive homogeneous polynomilas and orthosymmetric multilinear mappings which leads to a constructively proofs of **Sundaresan** results.

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- **1991 : Sundaresan** On ℓ_p and $L_p[0,1]$

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- **2005 : Garcia , Villaneva, Carando, Lassale, Zalduendo** : On $C(X)$

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- A bilinear map $T : E \times E \to F$ is positive if $T(x, y) \geq 0$ whenever $(x, y) \in E^+ \times E^+$, and is order bounded if given $(x, y) \in E^+ \times E^+$ there exists $a \in F^+$ such that $|T(z, w)| \leq a$ for all $(0, 0) \leq (z, w) \leq (x, y) \in E \times E$
- $T : E \times E \to F$ is (r-u) continuous if $x_n, y_n \longrightarrow 0$ (r-u) in E implies that $T(x_n, y_n) \longrightarrow 0$ (r-u) in F.

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- The set $\mathcal{L}_b(E)$ of all order bounded operators on E is an ordered vector space with respect to the pointwise operations and order. The positive cone of $\mathcal{L}_b(E)$ is the subset of all positive operators.
- An element T in $\mathcal{L}_b(E)$ is referred to as an orthomorphism if, for all $x, y \in E$, $|T(x)| \wedge |y| = 0$ whenever $|x| \wedge |y| = 0$. Under the ordering and operations inherited from $\mathcal{L}_b(E)$, the set $Orth(E)$ of all orthomorphisms on E is an Archimedean Riesz space.
- The Riesz algebra E is said to be an f-algebra whenever $x \wedge y = 0$ then $xz \wedge y = zx \wedge y = 0$ for all $z \in E^+$.
- If E is a Riesz space then the Riesz space $Orth(E)$ is an f-algebra with respect to the composition as multiplication. Moreover the identity map on E is the multiplicative unit of $Orth(E)$. In particular, the f-algebra $Orth(E)$ is semiprime and commutative.
- If E is an f-algebra with unit element, then the mapping $\pi : x \longrightarrow \pi_x$ from E into $Orth(E)$ is a Riesz and algebra isomorphism, where $\pi_x(y) = xy$ for all $y \in E$.

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- A Dedekind complete Riesz space E is said to be *universally complete* whenever every set of pairwise disjoint positive elements has a supremum.
- Every Archimedean Riesz space E has a unique (up to a Riesz isomorphism) universally completion denoted E^u, ie., there exists a unique universally complete Riesz space such that E can be identified with an order dense Riesz subspace of E^u.
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Theorem

Let E be a relatively uniformly complete Riesz spaces, F be a Hausdorff t.v.s. (not necessarily a Riesz spaces) and let $\varphi : E \times E \longrightarrow F$ be a continuous orthosymmetric bilinear map then φ is symmetric

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Let E be a Riesz space, F be a Hausdorff t.v.s., and let $T : E^n \longrightarrow F$ be a continuous orthosymmetric multilinear map such that $(T(E^n))'$ separates points. If $\sigma \in S(n)$ is a permutation then

$$T(x_1, ..., x_n) = T(x_{\sigma(1)}, ..., x_{\sigma(n)})$$

for all $x_1, ..., x_n \in E$.

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$$T(\pi_1(x_1), ..., \pi_n(x_n)) = T(x_1, ..., \pi_1...\pi_n(x_n))$$

for all $x_1, ..., x_n \in E$ and $\pi_1, ..., \pi_n \in Orth(E)$.

orthogonally additive homogeneous polynomials

- Let E be a vector lattice and let F be a topological space. A map $P : E \to F$ is called a homogeneous polynomial of degree n (or a n-homogeneous polynomial) if $P(x) = T(x, .., x)$, where T is a n-multilinear map from E^n into F.
- A homogeneous polynomial, of degree n, $P : E \to F$ is said to be orthogonally additive if $P(x + y) = P(x) + P(y)$ where $x, y \in E$ are orthogonally (i.e. $|x| \wedge |y| = 0$).
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Let E be an Archimedean vector lattice, F be a Hausdorff topological vector space (not necessarily a vector lattice), $T : E^n \to F$ be a (ru)-continuous orthosymmetric multilinear map such that $T(E^n)'$ separates points. Then there exists a linear operator $T_P : \overset{n}{\underset{i=1}{\pi}} E^{ru} \to F$ such that

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Structure Problem

Let E be an Archimedean vector lattice, F be a Hausdorff topological vector space (not necessarily a vector lattice) and let $P \in \mathcal{P}_{C0}({}^nE, F)$ whose associated symmetric multilinear map T satisfies $T(E^n)'$ separates points. Then there exists a linear operator $T_P : \underset{i=1}{\overset{n}{\pi}} E^{ru} \to F$ such that

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Our approach fails for the non continuous case

Structure Problem

Let E be the Riesz space of all real valued functions f on $[0,1]$ satisfying that there is a finite subset $(x_i)_{1\leq i\leq n}$ such that $0 = x_0 < x_1 < .. < x_n = 1$ and on each interval $[x_{i-1}, x_i)$ $f(x) = m_i(f)x + b_i(f)$ and $T(f,g) = m_0(f)b_0(g)$.

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Polymorphisms and Polyorthomorphisms

- $P \in \mathcal{P}_b(^nE, F)$ with F Dedekind complete $|P|(x) = |T_p|(x^n)$for all $x \in E^+$
- $\mathcal{P}_b(^nE, F^\delta)$ is a Riesz space.
- F Dedekind complete then $\mathcal{P}_b(^nE, F) \sim \mathcal{L}_b(\underset{i=1}{\overset{n}{\pi}} E^{ru}, F)$
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- G Riesz subspace of E, F Dedekind complet, $P \in \mathcal{P}_b({}^nE, F)$, $Q \in \mathcal{P}_b({}^nG, F)$, $|Q| \leq P$ then $Q \in \mathcal{P}_b({}^nE, F)$
- $\mathcal{P}_b({}^nE, F) \subset \mathcal{P}_b({}^nE^\delta, F)$
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Let E and F be two Riesz spaces. A mapping $T : E \longrightarrow F$ is said to be a *lattice homomorphism* whenever

$$T(x \vee y) = Tx \vee Ty \text{ and } T(x \wedge y) = Tx \wedge Ty, \text{ for all } x, y \in E.$$

A linear homomorphism is called a *Riesz homomorphism.*

The study of the relationship between lattice and Riesz homomorphism was really inaugurated in 1978 thanks to the fundamental work of Mena and Roth, who were interested essentially for the setting of the Riesz spaces of real valued continuous functions defined on a compact Hausdorff spaces. They proved that if X and Y are compact Hausdorff spaces and $T : C(X) \to C(Y)$ is a lattice homomorphism such that $T(\lambda 1) = \lambda T(1)$ for all $\lambda \in \mathbb{R}$, then T is linear. Later, many authors interested in this problem. Thanh generalized Mena and Roth's result to the case when X and Y are real compact spaces. There are another generalization by Lochan and Strauss . So far the best results, in this field, are duo to Ercan and Wickstead. They showed from the theorem of Mana and Roth by using the Kakutani representation theorem, that if E and F are uniformly complete Archimedean Riesz spaces with weak order units $e_1 \in E$ and $e_2 \in F$ and if $T : E \to F$ is a lattice homomorphism such that $T(\lambda e_1) = \lambda e_2$ for all $\lambda \in \mathbb{R}$, then T is linear. As an application they gave a corresponding result for the case where E and F are two uniformly complete Archimedean Riesz spaces with disjoint complete systems of projections as an example for two σ-Dedekind complete Riesz spaces.

- The topological richness of the uniformly complete structure of Riesz spaces with weak order units which gives a particular interest to lattice homomorphisms on those spaces. It seems natural therefore to ask
- What happenings in the general case of Riesz spaces ? What about weaker condition under which a lattice homomorphism defined on a Riesz space is linear ?

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- What happenings in the general case of Riesz spaces ? What about weaker condition under which a lattice homomorphism defined on a Riesz space is linear ?

- A prime ideal P of a Riesz space E is a nonempty proper lattice subset of E (not necessarily a vector subspace) that containing with any element all smaller ones and with $x \in P$ or $y \in P$ whenever $x \wedge y \in P$
- We say that a prime ideal P in $C(X)$ is associated with a point $x \in X$ if $g \in P$ whenever $f \in P$ and $g(x) < f(x)$.
- For X compact, every prime ideal in $C(X)$ is associated with some point of X and this point is unique if X is also Hausdorff
- If $P \subset Q$ where P and Q are prime ideals in $C(X)$, X is a compact Hausdorff space then P and Q are associated with the same point

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- Let E be a Riesz space and P be a prime ideal of E then P^{ru}, the uniform completion of P, is a prime ideal of E^{ru}
- Let E be a uniformly dense Riesz subspace of $C(X)$ where X is a compact Haudorff space and P a prime ideal. Then P and P^{ru} are associated with the same point $x \in X$.

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Let E be a uniformly dense Riesz subspace of $C(X)$ where X is a compact Haudorff space such that $1 \in E$. Let $\phi : E \to \mathbb{R}$ be a lattice homomorphism satisfying $\phi(\lambda) = \lambda$ for all $\lambda \in \mathbb{R}$. Then there exist $x \in X$ such that $\phi = \delta_x$, the point evaluation map defined for $f \in C(X)$ by $\delta_x(f) = f(x)$.

- Let E be a Riesz space with strong order unit e and let F be a Riesz space. If T is a lattice homomorphism from E into F such that $T(\lambda e) = \lambda T(e)$ for each $\lambda \in \mathbb{R}$ then T is linear (Riesz homomorphism).
- Let E and F be Riesz spaces. If T is a lattice homomorphism from E into F such that $T(\lambda x) = \lambda T(x)$ for each $\lambda \in \mathbb{R}$ and $x \in E$ then T is linear (Riesz homomorphism).
- Let E and F be a Riesz spaces with weak order units e and f, respectively. If T is a lattice homomorphism from E into F satisfying $T(\lambda e) = \lambda f$ for each $\lambda \in \mathbb{R}$ then T is a Riesz homomorphism (linear).
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- Let E be a Riesz space with strong order unit e and let F be a Riesz space. If T is a lattice homomorphism from E into F such that $T(\lambda e) = \lambda T(e)$ for each $\lambda \in \mathbb{R}$ then T is linear (Riesz homomorphism).
- Let E and F be Riesz spaces. If T is a lattice homomorphism from E into F such that $T(\lambda x) = \lambda T(x)$ for each $\lambda \in \mathbb{R}$ and $x \in E$ then T is linear (Riesz homomorphism).
- Let E and F be a Riesz spaces with weak order units e and f, respectively. If T is a lattice homomorphism from E into F satisfying $T(\lambda e) = \lambda f$ for each $\lambda \in \mathbb{R}$ then T is a Riesz homomorphism (linear).
- Let E and F be Riesz spaces with a weak order unit e of E. If T is a lattice isomorphism from E into F such that $T(\lambda e) = \lambda T(e)$ for each $\lambda \in \mathbb{R}$ then T is a Riesz isomorphism (linear).

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Let (X, d) and (Y, d') be complete metric spaces and $T : Lip(X) \rightarrow Lip(Y)$ be a lattice isomorphism with $T(\lambda 1) = \lambda T(1)$ for each $\lambda \in \mathbb{R}$. Then X and Y are lipschitz homeomorphic.

Let

$$X = \bigcup_{n\geq 2}\left\{n, n+\frac{1}{n}\right\} \text{ and } Y = \bigcup_{n\geq 2}\left\{n^2, n^2+1\right\}$$

equipped with the usual metric in $\mathbb{R}$. Then X and Y are two complete metric spaces. Now consider the map $T : Lip(X) \rightarrow Lip(Y)$ defined by

$$T(f)(n^2) = nf(n) \text{ and } T(f)(n^2+1) = nf(n+\frac{1}{n}).$$

Then T defines a lattice isomorphism between $Lip(X)$ and $Lip(Y)$ but X and Y are non lipschitz homeomorphic complete metric spaces

THANK YOU FOR YOUR ATTENTION