Generalized Ergodic Domination in Ordered Banach Algebras

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Outline

1. Preliminaries
   - Inessential ideals
   - Riesz points
   - Ordered Banach algebras
   - Domination problem
2. Generalized Domination in OBAs
   - The Generalized Domination Problem
   - Some definitions
   - Some preliminary results
3. Generalized ergodic domination in OBAs
   - Generalized Ergodic Domination Theorem
4. Banach lattice algebras
   - Generalized weak monotonicity of the spectral radius
   - Applications to regular operators

Notations

- E: complex Banach lattice, i.e

$$E = E_{\mathbb{R}} + iE_{\mathbb{R}} = \{x + iy : x, y \in E_{\mathbb{R}}\}.$$

- $\mathcal{L}(E)$: bounded linear operators on E.
- $\mathcal{L}^+(E)$: the set of all positive operators on E.
- $\mathcal{L}^r(E)$: regular operators on E i.e. linear combinations of positive operators.
- $\mathcal{K}^r(E)$: closure in $\mathcal{L}^r(E)$ of the finite rank operators on E (the r-compact operators).

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- A: complex unital Banach algebra.
- $\sigma(a)$ $(a \in A)$: the spectrum of a.
- $r(a)$ $(a \in A)$: the spectral radius of a.
- $R_a : \mathbb{C} \setminus \sigma(a) \to A$, $R_a(\lambda) := (\lambda 1 - a)^{-1}$ $(a \in A)$: the resolvent function of a.

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Note: If E is Dedekind complete, then $\mathcal{L}^r(E)$ is a (Dedekind complete) Banach lattice.

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Example 1.2.

1. $\mathrm{Rad}(A)$ (the Jacobson radical of A),
2. $\mathcal{K}(E)$ (the compact operators) in $\mathcal{L}(E)$
3. $\mathcal{F}(E)$ (the finite rank operators) in $\mathcal{L}(E)$
4. $\mathcal{K}^r(E)$ in $\mathcal{L}^r(E)$ [Arendt, 1981, Theorem 2.6][a].

[a] Arendt, W. (1981). On the o-spektrum of regular operators and the spectrum of measures. *Mathematische Zeitschrift*, 178(2):271–287

Riesz points

Definition 1.3 (Riesz points).

If $a \in A$, $\lambda \in \text{iso}\, \sigma(a)$ is a *Riesz point* of $\sigma(a)$ (relative to an ideal I) if $p(\lambda, a) \in I$ with

$$p(\lambda, a) := \frac{1}{2\pi i} \int_\Gamma R_a(\alpha) \mathrm{d}\alpha,$$

where Γ is a contour surrounding and isolating λ in $\sigma(a)$.

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- If A is semisimple, then λ is a Riesz point of $\sigma(a)$ if and only if λ is a pole of R_a and $p(\lambda, a) \in I$ [Mouton, 2002][1].

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- $\mathcal{L}(E), \mathcal{L}^r(E)$ are semisimple (well known).

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Ordered Banach algebras

Definition 1.4 (Algebra cone – Ordered Banach algebra).

An *algebra cone* is a non-empty subset $C \subseteq A$ s.t.

1. $1 \in C$, (contains the unit)
2. $C + C \subseteq C$, (closed under addition)
3. $\lambda C \subseteq C$ ($\lambda \in \mathbb{R}^+$), (closed under non-negative scalar multiplication)
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Example 1.5.

1. $(\mathbb{C}, \mathbb{R}^+)$, $(\mathcal{M}_n(\mathbb{C}), \mathcal{M}_n(\mathbb{R}^+))$.
2. $(\mathcal{L}(E), \mathcal{L}^+(E))$, $(\mathcal{L}^r(E), \mathcal{L}^+(E))$.
3. $(A/I, \pi C)$, where (A, C) is an OBA, I a closed ideal (e.g. $(\mathcal{L}^r(E)/\mathcal{K}^r(E), \pi\mathcal{L}^+(E))$), where $\pi : A \to A/I$ is the canonical homomorphism.

Ordered Banach algebras (Continued)

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Definition 1.6 (Normality of C – (Weak) Monotonicity of r).

- C is **normal** if $\exists\alpha > 0$ (called *a normality constant*) such that $a, b \in A,\ 0 \leq a \leq b \implies \|a\| \leq \alpha \|b\|$.

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- OBAs with normal algebra cone: $(\mathbb{C}, \mathbb{R}^+)$, $(\mathcal{M}_n(\mathbb{C}), \mathcal{M}_n(\mathbb{R}^+))$, $(\mathcal{L}(E), \mathcal{L}^+(E))$, $(\mathcal{L}^r(E), \mathcal{L}^+(E))$, ...

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- If C is normal, then r monotone in (A, C) [Raubenheimer and Rode, 1996][a].

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- If C is normal, then r monotone in (A, C) [Raubenheimer and Rode, 1996][a].
- r is weakly monotone in $\mathcal{L}^r(E)/\mathcal{K}^r(E)$ [Martínez and Mazón, 1991][b].

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[a] Raubenheimer, H. and Rode, S. (1996). Cones in Banach algebras. *Indagationes Mathematicae (N.S.)*, 7(4):489–502

[b] Martínez, J. and Mazón, J. M. (1991). Quasi-compactness of dominated positive operators and C_0-semigroups. *Mathematische Zeitschrift*, 207(1):109–120

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Theorem 1.9 ([Räbiger and Wolff, 1997]).

Let $S, T \in \mathcal{L}(E)$ such that $\pm S \leq T$. If T is uniformly ergodic with finite rank projection, then so is S.

Note: $\pm S \leq T$ iff $T \pm S \geq 0$ iff $|Sx| \leq T\,|x|$ for all $x \in E$.

The Generalized Domination Problem

Definition 2.1 (Ergodic elements).

An element $a \in A$ is *ergodic* if the sequence $\left(\frac{1}{n}\sum_{k=0}^{n-1} a^k\right)$ converges in A.

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- **Domination by ergodic elements in OBAs:**

Theorem 2.2 (Ergodic Domination: [Mouton and Muzundu, 2014]).

Let (A, C) be a semisimple OBA with C closed and normal, and I a closed inessential ideal of A such that r is weakly monotone in A/I. Suppose that $a, b \in A$ such that $0 \leq a \leq b$ and $r(b)$ is a Riesz point of $\sigma(b)$. If b is ergodic, then a is ergodic.

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- **An open question in [Mouton and Raubenheimer, 2017]**[2]**:**
  Can Theorem 2.2 be extended by replacing the condition "$0 \leq a \leq b$ with the weaker condition "$\pm a \leq b$" ?

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Generalized normality – Generalized monotonicity

Let (A, C) be an OBA and I a closed ideal of A.

Definition 2.4.

- C is **generalized normal** if $\exists\alpha > 0$ such that

$$a, b \in A, \ \pm a \leq b \implies \|a\| \leq \alpha \|b\| .$$

- r is **generalized monotone in** (A, C) if $\pm a \leq b \implies r(a) \leq r(b)$.
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Some obvious implications:

1. Generalized normality $\implies$ normality.
2. Generalized monotonicity $\implies$ monotonicity.
3. Generalized weak monotonicity $\implies$ weak monotonicity.

Generalized normality

With the same argument as in [Raubenheimer and Rode, 1996][3]:

Proposition.

If C is generalized normal, then r is generalized monotone in (A, C).

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If (A, C) is an OBA, then C is generalized normal if and only if C is normal.

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Corollary 2.6.

If C is normal, then r is ***generalized*** *monotone.*

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Generalized domination of ergodic elements

An extension of [Mouton and Muzundu, 2014, Theorem 5.2][4]:

Theorem 3.1.

Let (A, C) be an OBA with C normal and $a, b \in A$ such that $\pm a \leq b$. Suppose that either $r(a) \notin \sigma(a)$ or 1 is a pole of R_a. If b is ergodic, then a is ergodic.

Main tools for the proof.

- The fact that normality of C implies generalized monotonicity of r.
- The Ergodic Theorem [Mouton and Muzundu, 2014].

□

[4] Mouton, S. and Muzundu, K. (2014). Domination by ergodic elements in ordered Banach algebras. *Positivity*, 18(1):119–130

Generalized Ergodic Domination Theorem

An answer to the open question in [Mouton and Raubenheimer, 2017][5]:

Theorem 3.2.

Let (A, C) be a semisimple OBA with C closed and normal, and I a closed inessential ideal of A such that the spectral radius is generalized weakly monotone in A/I. Suppose that $a, b \in A$ such that $\pm a \leq b$ and $r(b)$ is a Riesz point of $\sigma(b)$. If b is ergodic, then a is ergodic.

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Generalized weak monotonicity of r in BLAs

Definition 4.1 (Banach lattice algebra).

A Banach algebra A which is also a Banach lattice and satisfying:

1. $|ab| \leq |a|\,|b|$ for all $a, b \in A$.

Definition 4.2 (m-order ideal ($m \in \mathbb{N}$)).

An (algebraic) ideal I of a Banach lattice algebra A such that:

1. If $a \in I$, then $|a| \in I$.
2. If $a, b \in A$ such that $|a| \leq b$ and $b \in I$, then $a^m \in I$.

[6] Koumba, U. and Raubenheimer, H. (2015). Positive Riesz operators. *Mathematical Proceedings of the Royal Irish Academy*, 115(1):1–11

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An extension of [Koumba and Raubenheimer, 2015, Theorem 3.8][6]:

Theorem 4.3.

Let A be a Banach lattice algebra, and I a closed inessential m-order ideal of A. If $a, b \in A$ such that $\pm a \leq b$, then $r(a + I) \leq r(b + I)$.

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Applications to regular operators

If E is Dedekind complete, then $\mathcal{L}^r(E)$ is a Banach lattice algebra and $\mathcal{K}^r(E)$ is a closed inessential 3-order ideal.

[7] Martínez, J. and Mazón, J. M. (1991). Quasi-compactness of dominated positive operators and C_0-semigroups.
Mathematische Zeitschrift, 207(1):109–120

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If E is Dedekind complete, then $\mathcal{L}^r(E)$ is a Banach lattice algebra and $\mathcal{K}^r(E)$ is a closed inessential 3-order ideal.Hence, we have:

- A generalizaton of [Martínez and Mazón, 1991, Theorem 2.8][7]:

Corollary 4.4.

If E is Dedekind complete, then r is generalized weakly monotone in $\mathcal{L}^r(E)/\mathcal{K}^r(E)$, i.e, for all $S, T \in \mathcal{L}^r(E)$, such that $\pm S \leq T$, we have that

$$r(S + \mathcal{K}^r(E)) \leq r(T + \mathcal{K}^r(E)).$$

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$$r(S + \mathcal{K}^r(E)) \leq r(T + \mathcal{K}^r(E)).$$

- A special case of Räbiger and Wolff's result, proved free of operator theory:

Theorem 4.5.

Let E be a Dedekind complete Banach lattice and $S, T \in \mathcal{L}^r(E)$ such that $\pm S \leq T$ and $r(T)$ is a Riesz point of $\sigma(T, \mathcal{L}^r(E))$. If T is r-ergodic, then S is r-ergodic (where r-ergodic means ergodic in $\mathcal{L}^r(E)$).

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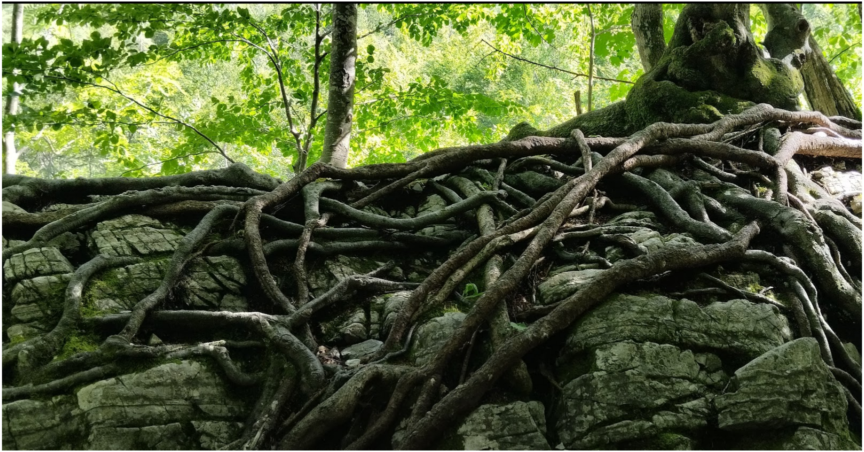

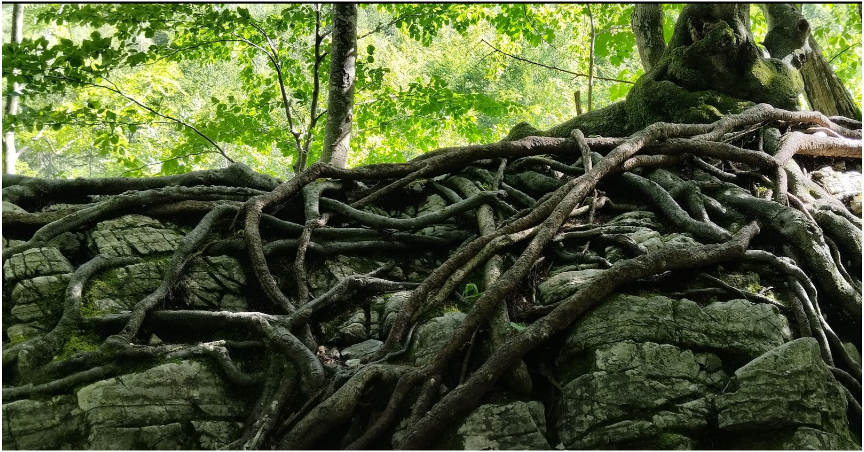

Thanks for your attention!