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Edited by Lucijan Plevnik

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University of Ljubljana
Faculty of Mathematics and Physics

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Contents

Plenary talks

Truncated vector lattices: Something old and something new

Karim Boulabiar
University of Tunis El Manar, Tunisia
karim.boulabiar@fst.utm.tn

Truncated vector lattices of functions has been introduced by Marshall Stone as the natural framework for pre-integrals to be represented as integrals. Later, David Fremlin obtained a representation of certain abstract normed vector lattices as truncated vector lattices of bounded functions (by the way, the terminology "truncated vector lattices" is due to Fremlin). In the mid-eighties, in the last century, Charles Boudewijn Huijsmans and Ben de Pagter focused their attention on truncated functions algebras in the sense of Birkhoff and Pierce. About ten years ago, Richard Ball gave a remarkable intrinsic axiomatization of this concept. Over the past five years, the Tunisian school made a significant contribution on this old-new theory. In this talk, we intend to tell the story of the theory of truncated vector lattices from its birth to today.

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Metric geometry on symmetric cones

Bas Lemmens
University of Kent, United Kingdom
b.lemmens@kent.ac.uk

The interior of a solid closed cone C in an inner-product space is said to be a symmetric cone if C is self-dual and the group of linear automorphisms of C acts transitively on the interior. These cones are in one-to-one correspondence with the interiors of the cones of squares in Euclidean Jordan algebras by the Koecher-Vinberg theorem. They also play an important role real differential geometry where they are used to realise a variety of non-compact type Riemannian symmetric spaces. Besides the Riemannian metric on the these cones, there are other important metrics on cones that are defined in a purely order theoretic way, such as the Hilbert metric and Thompson metric. These metrics have a Finsler structure. In this talk I will give an over view of the metric geometry of these metrics on symmetric cones. In particular, I will discuss the geometry and global topology of the horofunction compactification of these spaces. We will see that these compactifications admit a concrete realisation as the closed dual unit ball of the Finsler norm in the tangent space.

The talk is partly based on joint work with Kieran Power.

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Embeddability of real and positive operators

Agnes Radl
University of Applied Sciences Fulda, Germany
agnes.radl@informatik.hs-fulda.de

It is a well-known problem in probability theory whether a Markov matrix is embeddable into a Markov semigroup. Even today it is an active field of research, see e. g. the recent survey [1]. We consider a related problem: Given a (finite or infinite) matrix T, is it embeddable into a real/positive C_0-semigroup, i. e., is there a real/positive C_0-semigroup $(T(t))_{t\geq 0}$ such that $T(1) = T$?

We will give necessary and sufficient conditions for embeddability of a real matrix into a real C_0-semigroup. Moreover, we will see that real-embeddability is typical for real contractions on ℓ^2.

In the case that T is positive we will present necessary conditions for embeddability of T into a positive C_0-semigroup. In addition, we will give a full description of embeddability for positive 2×2 matrices.

The talk is based on joint work with Tanja Eisner [2].

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Positivity and well-posedness of a class of linear systems

Abdelaziz Rhandi

University of Salerno, Italy

arhandi@unisa.it

Starting from Greiner's work [3] on perturbing the boundary conditions of a generator and its generalization to unbounded boundary perturbations in [4], we provide in this talk useful criteria for positivity and well-posedness of a class of infinite-dimensional control systems. These criteria are based on an inverse estimate with respect to the Hille-Yosida Theorem. Indeed, we establish a generation result for perturbed positive operator semigroups. This unifies previous results available in the literature and that were established separately so far, see [1, 2, 5, 6]. Furthermore, applications to a Boltzmann equation on a finite network and size-dependent population system with delayed birth process are also presented.

The talk is based on joint work with Y. El Gantouh.

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From Loewner's characterization of operator monotone functions to fundamental theorem of chronogeometry

Peter Šemrl

Institute of Mathematics, Physics and Mechanics, Slovenia

peter.semrl@fmf.uni-lj.si

Local order isomorphisms of matrix and operator domains will be discussed. A connection with Loewner's theorem and the fundamental theorem of chronogeometry will be explained. The first one characterizes operator monotone functions while the second one describes the general form of bijective preservers of light-likeness on the classical Minkowski space.

The talk is based on joint work with Michiya Mori.

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Recent progress in Banach lattices

Pedro Tradacete
Instituto de Ciencias Matemáticas (CSIC), Spain
pedro.tradacete@icmat.es

We will survey recent results in the theory Banach lattices, focussing mainly on the interactions with Banach space theory. In particular, we will discuss the relations between linear and lattice embeddings, complemented subspaces of Banach lattices and some recent tools like the free Banach lattice generated by a Banach space.

Convergence structures in vector lattices

Vladimir Troitsky
University of Alberta, Canada
troitsky@ualberta.ca

Several important convergences that appear naturally in vector lattice theory are not topological. These include order, unbounded order, and relative uniform convergences of nets. However, they fit into the framework of the theory of convergence structures. This theory was originally formulated in terms of filters, which made it impractical for applications in analysis. However, it was recently reformulated in the language of nets.

In the talk, we will present an overview of the theory of convergence structures in terms of nets. We will revisit order, unbounded order, and relative uniform convergences in vector lattices from the point of view of convergence structures. We will discuss locally solid convergence structures; this concept is the natural extension of a locally solid topology to the framework of convergence structures.

Direct and inverse limits of vector lattices

Jan Harm van der Walt
University of Pretoria, South Africa
janharm.vanderwalt@up.ac.za

Direct and inverse limits are useful constructions occurring frequently in analysis, but have received very little attention in the context of vector lattices. We discuss the existence and properties of such limits in suitable categories of vector lattices. The main results are duality theorems which state, broadly speaking, that the order (continuous) dual of the direct limit of a direct system of vector lattices is the inverse limit of the order (continuous) duals of the component spaces of the original system. Similar results hold for the order (continuous) dual of the inverse limit of an inverse system of vector lattices. We present some applications of these results, including a representation theorem for the order bidual of $\mathrm{C}(X)$ for a realcompact space X.

The talk is based on joint work with Marcel de Jeu and Walt van Amstel.

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The order center and the algebraic center of JB-algebras

ONNO VAN GAANS
Leiden University, Netherlands
vangaans@math.leidenuniv.nl

For a directed partially ordered vector space X, one can consider the vector space of all linear operators on X ordered by the cone of positive operators. The subspace of all operators that are below and above a multiple of the identity operator is called the *order center* of X. By the algebraic center of an associative algebra, one usually means the set of those elements that commute with all other elements.

We will consider JB-algebras, which are both Jordan algebras and Banach spaces with suitably compatible norms. A typical example is the vector space of all self-adjoint operators on a Hilbert space with the Jordan product $A \circ B = \frac{1}{2}(AB + BA)$. A JB-algebra is commutative but typically not associative. Endowed with the cone of squares, it is a directed partially ordered vector space. The *algebraic center* of a JB-algebra X is defined to be the set of those elements whose corresponding left multiplication operator commutes with all other left multiplication operators. We will show that the order center and the algebraic center of a unital JB-algebra are isomorphic.

The talk is based on joint work with Anke Kalauch (TU Dresden) and Mark Roelands (Leiden University).

Continuity and robustness of risk measures

FOIVOS XANTHOS
Toronto Metropolitan University, Canada
foivos@torontomu.ca

A milestone result in the theory of Banach lattices asserts that any positive operator between Banach lattices is continuous. In this talk we will discuss variations of this result and applications to statistical robustness of risk measures.

The talk is based on joint work with Niushan Gao and Cosimo Munari.

CA18232: Mathematical models for interacting dynamics on networks

Talks in this session are based upon work from COST Action Mathematical models for interacting dynamics on networks, CA18232 (`www.mat-dyn-net.eu`), supported by COST (European Cooperation in Science and Technology). COST (`www.cost.eu`) is a funding agency for research and innovation networks. Their Actions help connect research initiatives across Europe and enable scientists to grow their ideas by sharing them with their peers. This boosts their research, career and innovation.

Irreducibility of eventually positive semigroups

Sahiba Arora
Dresden University of Technology, Germany
`sahiba.arora@mailbox.tu-dresden.de`

Positive operator semigroups that occur in concrete applications are quite often irreducible, which is why a deep and extensive theory of irreducibility has been developed. Various arguments from this theory, however, break down if the semigroup is only eventually positive – a property that has recently been shown to occur in numerous concrete evolutions equations. This necessitates the introduction of tools that also work for the eventual positivity case. Indeed, the lack of positivity for small times makes it necessary to consider ideals that might only be invariant for large times. This leads us to, what we call, the *persistently irreducible* semigroups.

In this talk, we introduce persistently irreducible semigroups and look at some of their properties in the setting of eventual positivity.

The talk is based on joint work with Jochen Glück.

Notes on oscillating semigroups

András Bátkai
Vorarlberg University of Education, Austria
`andras.batkai@ph-vorarlberg.ac.at`

Oscillating semigroups in a Banach lattice have been investigated in detail by Stroinski [1, 2]. He generalized this notion from oscillating differential equations, arriving to various possible notions.

In this talk, we review this notion, present some perturbation results, and give applications to more recent problems.

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Positivity and well-posedness properties of a class of linear control systems

Yassine El Gantouh
Zhejiang Normal University, China
elgantouhyassine@gmail.com

In many practical applications of control theory some constraints on the state and/or on the control need to be imposed. This is for instance the case of evolutionary networks (heat conduction, transportation networks, population dynamics etc.) where realistic models have to take into account that the state and/or the control represents some physical quantity which must necessarily remain positive.

In this talk, the positivity and well-posedness properties of infinite-dimensional linear systems with unbounded input and output operators are highlighted. Furthermore, an applications to a Boltzmann equation on a finite network is also presented.

The talk is based on the following works.

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Order continuity and maximal inequalities for C_0-semigroups

Jochen Glück
University of Wuppertal, Germany
glueck@uni-wuppertal.de

For a positive C_0-semigroup $T = (T(t))_{t\in[0,\infty)}$ on a Banach lattice we discuss under which conditions every orbit is order continuous (or, equivalently, relatively uniformly continuous) with respect to the time parameter. It turns out that this equivalent to all orbits of the semigroup being order bounded for small times.

In the terminology of harmonic analysis, the latter property means that the semigroup operators satisfy a maximal inequality for small times. This property has surprisingly strong consequences: if, e.g., the underlying Banach lattice is an L^p-space, such a maximal inequality implies that the semigroup is analytic.

Parts of this talk are based on joint work with Michael Kaplin.

Eventual Cone Invariance Revisited

Julian Hölz
University of Wuppertal, Germany
hoelz@uni-wuppertal.de

We consider finite-dimensional real vector spaces X ordered by a closed cone X_+ with non-empty interior. In this talk we take a closer look at the following two notions of eventual nonnegativity: A matrix semigroup $(e^{tA})_{t\geq 0}$ is said to be (i) *uniformly eventually nonnegative*, if there exists a time t_0 such that $e^{tA}X_+ \subseteq X_+$ for all times $t \geq t_0$; (ii) *individually eventually nonnegative* if $e^{tA}x_0 \in X_+$ for all $x_0 \in X_+$ and all $t \geq t_0$, where the time t_0 may depend on the initial value x_0.

We show, by means of an example, that individual eventual nonnegativity does not imply uniform eventual nonnegativity.

The talk is based on joint work with Jochen Glück.

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Some results on spectral theory for suprema preserving operators on max-cones in normed vector lattices

Aljoša Peperko
University of Ljubljana, Slovenia
aljosa.peperko@fs.uni-lj.si

Suprema preserving nonlinear operators are interesting from the applicative and theorethical point of view. Although such operators may usually be sensibly defined on the whole normed vector or Banach lattice, they are often well behaved (suprema preserving, positively homogeneous, Lipschitz, ...) on some smaller cone. In the talk some elements of the spectral theory for such operators will be presented. For example, the eigenproblem of such operators naturally arises in the asymptotic study of periodic solutions of a class of differential-delay equations. Some analogous results for cone linear operators on normal cones in normed spaces will be pointed out.

The talk is mainly based on the three joint papers with Vladimir Müller (Prague).

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Embeddability of real and positive operators

AGNES RADL

University of Applied Sciences Fulda, Germany

agnes.radl@informatik.hs-fulda.de

You can find the abstract for this talk in the section Plenary talks.

Positivity and well-posedness of a class of linear systems

ABDELAZIZ RHANDI

University of Salerno, Italy

arhandi@unisa.it

You can find the abstract for this talk in the section Plenary talks.

Elliptic and parabolic operators with unbounded polynomial coefficients

CRISTIAN TACELLI

University of Salerno, Italy

ctacelli@unisa.it

We study generation results and domain characterization of some elliptic operators of second and fourth order with unbounded coefficients with polynomial degree.

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On the weak solvability of second order difference scheme corresponding to system of sine-Gordon equations

OZGUR YILDIRIM

Yildiz Technical University, Turkey

ozgury@yildiz.edu.tr

In the present study the weak solvability of the second order of accuracy difference scheme corresponding to the nonlinear system of sine-Gordon equations which describes DNA dynamics is considered. An unconditionally stable second order difference scheme generated by the unbounded operator A^2 is presented. The weak solvability is studied in the space of distributions by making use of the variational methods. In order to verify theoretical statements numerical example that uses the finite difference scheme together with the fixed point method is presented with numerical results.

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Contributed talks

On different modes of order convergence and some applications

Kevin Abela
University of Malta, Malta
kevin.abela.11@um.edu.mt

Order convergence was studied extensively on partially ordered sets, lattices and Riesz spaces. In the literature one finds various definitions of order convergence [2, 4, 5, 6, 7, 8]. In our work, we compare these different convergences and the topologies they generate.

In Chetcuti et al [3], it was shown that the order topology and the σ-strong topology $s(M, M_*)$ coincide on the bounded parts of a σ-finite von Neumann algebra. This work relies heavily on the assumption of σ-finiteness. As an application of our work, we show that the σ-finite assumption is redundant when the algebra is assumed to be atomic.

For a semi-finite measure space, we compare various topologies that exist on L^∞. In our work, we compare the order topology to the norm and the Mackey topology. Furthermore, we show that if the measure space is σ-finite, the topology of convergence in measure coincides with the order topology when restricted to bounded parts of L^∞[1].

The talk is based on joint work with E. Chetcuti and H. Weber.

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On limited and almost limited operators

Safak Alpay
Middle East Technical University, Turkey
safak@metu.edu.tr

A bounded subset A of a Banach space X is called limited if each weak* null sequence in X^*, the dual, converges uniformly to zero on A. A bounded subset A of a Banach lattice E is said to be almost limited if each disjoint weak* null sequence (f_n) in E^* is uniformly null on A. Accordingly an operator $T : X \to E$ is called limited if $T(B_X)$ is limited in E where B_X is the closed unit ball of X [1]. Compact operators are limited. The canonical embedding of c_0 into l_∞ is limited but not compact. An operator $T : X \to E$ is called almost limited if $T(B_X)$ is an almost limited subset of E [2]. Limited operators are almost limited. The identity operator I^∞ of l^∞ is almost limited but not limited.

We study the interplay between (almost) limited operators with those of L(M)-weakly compact, almost L(M)-weakly compact, semicompact, aDPO, and weakly compact operators and Banach lattices serving as domain and range of these operators.

The talk is based on joint work with Svetlana Gorokhova.

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General diagonal process and applications

Youssef Azouzi
University of Tunis El Manar, Tunisia
josefazouzi@gmail.com

We present in this talk two versions of diagonal process. The first one concerns countable properties and it is constructive. The second is, however, more general but not constructive. As applications of these results we solve some problems concerning un-compact operators and order and unbounded order convergence. We show, in particular, that the set of un-compact operators from a Banach space to a Banach lattice is norm closed. We notice that our general result concerning diagonal process allows us to provide very simple proofs of several known results in functional analysis and topology. It can be applied to convergences that are not topological.

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Tensor product of Riesz subspaces with applications

Mohamed Amine Ben Amor
University of Carthage, Tunisia
mohamedamine.benamor@ipest.rnu.tn

In this presentation, our main focus will be on the Dedekind complete Riesz tensor product as defined by Grobler[1]. Specifically, we will examine the product of Riesz subspaces, particularly Ideals and principal bands. Our goal is to derive a Riesz space version of the Fubini Theorem and to establish a connection between conditional weak mixing and ergodicity through the tensor product of Riesz subspaces.

In this talk, I will be discussing the results of several collaborative research efforts. The first study was conducted in collaboration with Omer Gok and Damala Yaman from Yildiz Technical University in Turkey, while the second involved the contributions of Bruce Watson and Jonathan Homann from the University of the Witwatersrand in South Africa.

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Various characterizations of unbounded order convergence

Eugene Bilokopytov
University of Alberta, Canada
bilokopy@ualberta.ca

In this talk we will present several equivalent characterizations of the unbounded order (uo) convergence for net. We will apply these abstract characterizations to derive criteria for uo convergence of nets in spaces of continuous functions, as well as spaces of measurable functions. It is well known that for sequences uo convergence is equivalent to convergence almost everywhere (in the topological or measure-theoretic sense), and so the presented criteria may be viewed as possible definitions of convergence almost everywhere for nets.

A generalization of Riesz* homomorphisms in order unit spaces

Florian Boisen
Dresden University of Technology, Germany
florian.boisen@mailbox.tu-dresden.de

In pre-Riesz spaces, positive linear maps which can be uniquely extended to lattice homomorphisms between their respective Riesz completions – so-called Riesz* homomorphisms – are studied. A linear map T is a Riesz* homomorphism if and only if the inclusion $T[F^{\mathrm{u}\ell}] \subseteq T[F]^{\mathrm{u}\ell}$ is satisfied for all nonempty finite subsets F. They were introduced by van Haandel who wrongly assumed that it suffices to require the inclusion for sets which consist of at most two elements. We provide a counterexample and investigate the relation between these two notions while we concentrate on the case of positive linear functionals on order unit spaces.

The talk is based on joint work with Anke Kalauch, Janko Stennder, and Onno van Gaans.

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A Stone-Weierstrass type theorem for truncated vector lattices of functions

Sameh Bououn
University of Tunis El Manar, Tunisia
sameh.bououn@fst.utm.tn

A nonempty S of real-valued functions is said to be truncated if it contains with any function f its meet with the constant function 1. Very recently, Boulabiar and Hajji obtained a Stone-Weierstrass type theorem for a truncated vector sublattice L of $C_0(X)$, where X is a locally compact Hausdorff space. Relaying heavily on the multiplicative version of the classical Stone-Weierstrass theorem, they roved that if L separates the points of X and vanishes nowhere on X, then L is uniformly dense in $C_0(X)$. In this paper, we shall provide a new, intrinsic, and more natural proof of this result. We shall then apply the result to prove that any truncated vector lattices of bounded real-valued functions is essentially a $C_0(X)$-type space for some locally compact Hausdorff space X. This yields, in particular, that for any arbitrary topological space Y, a locally compact Hausdorff space X can be found such that $C_0(Y)$ and $C_0(X)$ are essentially the same, eliminating any reason for considering Banach lattices of continuous functions vanishing at infinity on other than locally compact Hausdorff spaces.

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The norm of the Cesàro operator minus the identity acting on decreasing sequences

Santiago Boza
Polytechnical University of Catalonia, Spain
santiago.boza@upc.edu

Recently, several authors have considered the problem of determining optimal norm inequalities for discrete Hardy-type operators (like Cesàro or Copson). In this talk, we will talk about sharp bounds for the norms of the difference of the Cesàro operator with either the identity or the shift operator, when they are restricted to the cone of decreasing sequences in ℓ^p (which is closely related to the previously mentioned estimates). We also address the case of weighted inequalities and find an interesting contrast between the norms of these two difference operators.

The talk is based on joint work with Javier Soria (Universidad Complutense de Madrid).

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Conditional indicators

Dorsaf Cherif
University of Tunis el Manar, Tunisia
Dorsaf.cherif@fst.utm.tn

We introduce a large class of (so-called) conditional indicators, on a complete probability space with respect to a sub σ-algebra. A conditional indicator is a positive mapping, which is not necessary linear, but may share common features with the conditional expectation, such as the tower property or the projection property. Several characterizations are formulated. Beyond the definitions, we provide some non trivial examples that are used in finance and may inspire new developments in the theory of operators on Riesz spaces.

The talk is based on joint work with Emmanuel Lepinette.

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Multilinear orthosymmetric continuous map and the connection between lattice and Riesz homomorphisms in Riesz spaces

Elmiloud Chil
University of Tunis, Institut Préparatoire aux Etudes d'Ingénieurs de Tunis, Tunisia
elmiloud.chil@ipeit.rnu.tn

We show that any multilinear orthosymmetric continuous map from an Archimedean Riesz space into a Hausdorff topological vector space is symmetric. Then, we introduce a new concept, namely that of polyorthomorphism of degree n, on a relatively uniformly complete f-algebra. We prove that, for a such Dedekind complete f-algebra E, the space $P_{orth}({}^nE)$ of all polyorthomorphisms of degree n is a Riesz space.

In the second part, we study the connection between lattice and Riesz homomorphisms in Riesz spaces. We prove, under a certain condition, that any lattice homomorphism on a Riesz space is a Riesz homomorphism. This fits with the type of results by Mena and Roth, Thanh, Lochan and Strauss and Ercan and Wickstead.

The talk is based on joint work with A, Dorai, and F, Mekdour.

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The Gleason metric and fibers of $\mathcal{H}^\infty(B_{c_0})$

Yun Sung Choi

Pohang University of Science and Technology, Korea

mathchoi@postech.ac.kr

Let $\mathcal{H}^\infty(B_{c_0})$ be the algebra of all bounded holomorphic functions on the open unit ball of c_0 and $\mathcal{M}(\mathcal{H}^\infty(B_{c_0}))$ the spectrum of $\mathcal{H}^\infty(B_{c_0})$. We prove that for any point z in the closed unit ball of ℓ_∞ there exists an analytic injection of the open ball B_{ℓ_∞} into the fiber of z in $\mathcal{M}(\mathcal{H}^\infty(B_{c_0}))$, which is an isometry from the Gleason metric of B_{ℓ_∞} to the Gleason metric of $\mathcal{M}(\mathcal{H}^\infty(B_{c_0}))$. We also show that, for some Banach spaces X, B_{ℓ_∞} can be analyticaly injected into the fiber $\mathcal{M}_z(\mathcal{H}^\infty(B_X))$ for every point $z \in B_X$.

The talk is based on joint work with Javier Falcó, Domingo García, Mingu Jung and Manuel Maestre.

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Relations between selected geometric properties on the positive cone of all nonnegative and decreasing elements of symmetric Banach spaces

Maciej Ciesielski
Poznań University of Technology, Poland
maciej.ciesielski@put.poznan.pl

Let $L^0 = L^0(I)$ be a set of all (equivalence classes of) extended real valued μ-measurable functions on $I = [0, \alpha)$, where $0 < \alpha \leq \infty$. For any $x \in L^0$ we define $d_x(\lambda) = \mu\{s : |x(s)| > \lambda\}$, $x^*(t) = \inf\left\{\lambda > 0 : d_x(\lambda) \leq t\right\}$, $x^{**}(t) = \frac{1}{t}\int_0^t x^*(s)ds$ for all $t > 0$. A (quasi-) Banach function space E is called a symmetric (quasi-) Banach space if for any $x \in L^0$, $y \in E$ where $d_x(\lambda) = d_y(\lambda)$, $\lambda > 0$ we have $x \in E$, $\|x\|_E = \|y\|_E$. The *Hardy-Littlewood-Pólya relation* $\prec$ is given for any x, y in $L^1 + L^\infty$ by

$$x \prec y \Leftrightarrow x^{**}(t) \leq y^{**}(t) \quad \text{for all} \quad t > 0.$$

A symmetric (quasi-)Banach space E is said to be *strictly K-monotone* ($E \in (SKM)$) if for any $x, y \in E$ where $x^* \neq y^*$, $x \prec y$ we have $\|x\|_E < \|y\|_E$. A symmetric (quasi-)Banach space E is called *K-order continuous* ($E \in (KOC)$), if for any $x \in E$ and $(x_n) \subset E$ such that $x_n \prec x$, $x_n^* \to 0$ a.e. we have $\|x_n\|_E \to 0$. A symmetric (quasi-)Banach space E is said to be *decreasing (resp. increasing) uniformly K-monotone*, shortly $E \in (DUKM)$ (resp. $E \in (IUKM)$) if for any $(x_n), (y_n) \subset E$ such that $x_{n+1} \prec x_n \prec y_n$ (resp. $x_n \prec y_n \prec y_{n+1}$) for each $n \in \mathbb{N}$ and $\lim_{n\to\infty} \|x_n\|_E = \lim_{n\to\infty} \|y_n\|_E$, we have $\lim_{n\to\infty} \|x_n^* - y_n^*\|_E = 0$. Let $k \in \mathbb{N}$, $k \geq 2$. We say that a (quasi-) Banach function space E is *fully k-rotund*, shortly E is FkR, if each sequence $(x_n) \subset S_E$ such that $\|\sum_{i=1}^k x_{n,i}\|_E \to k$ for any its k-subsequences $(x_{n,1}), (x_{n,2}), \cdots, (x_{n,k})$, is a Cauchy sequence. E is said to be *compactly fully k-rotund*, shortly E is $CFkR$, if each sequence $(x_n) \subset S_E$ such that $\|\sum_{i=1}^k x_{n,i}\|_E \to k$ for any its k-subsequences $(x_{n,1}), (x_{n,2}), \cdots, (x_{n,k})$, forms a relatively compact set. A point $x \in S_E$ is said to be a *point of local fully k-rotundity*, shortly x is a point of $LFkR$, (resp. a *point of compact local fully k-rotundity*, shortly x is a point of $CLFkR$) if for each sequence $(x_n) \subset S_E$ such that $\|x + \sum_{i=1}^k x_{n,i}\|_E \to k+1$ for any its k-subsequences $(x_{n,1})$,$(x_{n,2})$,$\cdots$, $(x_{n,k})$, we have x_n converges to x in E (resp. (x_n) forms a relatively compact set). A (quasi-)Banach function space E is called *locally fully k-rotund*, shortly E is $LFkR$, (resp. *compactly locally fully k-rotund*, shortly E is $CLFkR$) if every point $x \in S_E$ is a point of $LFkR$ (resp. a point of $CLFkR$)

First, we answer the crucial question whether the property locally fully k-rotundity can be investigated equivalently only on the cone E^d of all nonnegative and nonincreasing elements of E. Moreover, we show a connection between reflexivity of a symmetric Banach function space E and compactly fully k-rotundity of E^d the positive cone of all nonnegative and decreasing elements of E. Next, we present relations between fully k-rotundity of E^d and decreasing and increasing uniform K-monotonicity on E. Finally, we discuss characterizations of K-order continuity and uniform K-monotonicity in symmetric quasi-Banach spaces.

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Mixing convergences and bornologies on Riesz spaces

Jurie Conradie

University of Cape Town, South Africa

jurie.conradie@uct.ac.za

Generalized inductive limit topologies (sometimes called mixed topologies) occur surprisingly often in the theory of topolgical vector spaces; Grothedieck's completeness theorem is a case in point. In its most general form, such a topology is the finest vector topology coinciding with a given vector topology on all the sets of a bornology (a family of sets behaving like bounded sets).

In the theory of convergence spaces, there is a similar notion of an inductive limit, usually known as *specified sets convergence.* There are convergences, not derived from a topology, that are of this type. An example is order convergence in a Riesz space.

In this we develop a general framework for inductive limit convergence structures of this kind and use examples to illustrate the difference between the topological and the more general convergence setting. We also mention a number of open problems which arise naturally in this context.

The talk is based on joint work with Eugene Bilokopytov, Vladimir Troitsky and Jan Harm van der Walt.

The Complemented Subspace Problem in Banach lattices: A counterexample

David de Hevia

Instituto de Ciencias Matemáticas, Spain

david.dehevia@icmat.es

The Complemented Subspace Problem in Banach lattices asks whether any complemented subspace of a Banach lattice must be isomorphic to a Banach lattice. This question has several well-known ramifications in Banach space theory, for example, the following questions remain unanswered:

- Is every complemented subspace of $L_1[0,1]$ isomorphic to ℓ_1 or to $L_1[0,1]$?
- Is every complemented subspace of $C[0,1]$ isomorphic to a $C(K)$-space?
- Does every complemented subspace of a space with unconditional basis have unconditional basis?

In 2021, G. Plebanek and A. Salguero-Alarcón [2] provided an example, denoted by PS_2, of a one-complemented subspace of certain $C(K)$-space which is not isomorphic to any $C(L)$ (with L being an arbitrary compact Hausdorff space). This answered in the negative the Complemented Subspace Problem in the $C(K)$ setting. Recently, G. Martínez-Cervantes, A.

Salguero-Alarcón, P. Tradacete and I have checked that, in fact, the space PS_2 is not even isomorphic to a Banach lattice [1]. Nevertheless, since PS_2 is a non-separable Banach space, the separable case is still open.

The talk is based on joint work with G. Martínez-Cervantes, A. Salguero-Alarcón and P. Tradacete.

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Translation invariant Banach function spaces on locally compact groups

Chun Ding

Leiden University, Netherlands

c.ding@math.leidenuniv.nl

Let G be a locally compact group, and let E be a Banach function space on G that is invariant under left and right translations. In this talk, we consider regular operators on E that commute with right translations. If $C_c(G)$ is a dense subspace of E, and if such a commuting operator is positive, then one can show that it is given by convolution with a positive, possibly unbounded, measure. This generalizes the corresponding result by Brainerd and Edwards when $E = L^p(G)$ $(1 \leq p < \infty)$. We are also concerned with other properties of such spaces, such as the order continuity of the norm.
This work is partly inspired by unpublished results by Ben de Pagter and Werner Ricker for compact abelian groups.

The talk is based on an unpublished work.

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Free dual Banach lattices

Enrique García-Sánchez

Instituto de Ciencias Matemáticas, Spain

enrique.garcia@icmat.es

The free Banach lattice generated by a Banach space E is a Banach lattice $FBL[E]$, together with a linear isometric embedding $\phi_E : E \to FBL[E]$, satisfying the following universal property: for every Banach lattice X and every bounded and linear operator $T : E \to X$ there exists a unique lattice homomorphism $\hat{T} : FBL[E] \to X$ such that $\hat{T} \circ \phi_E = T$, with $\|\hat{T}\| = \|T\|$ [1]. Similarly, we can define the free p-convex Banach lattice, denoted $FBL^{(p)}[E]$, by imposing that the Banach lattices X in the previous definition is p-convex with p-convexity constant one [3]. At the same time, the bidual E^{**} of a Banach space E can be understood as a free object over E in the category of dual Banach spaces with adjoint operators. In this talk we will combine these two notions of free Banach lattices and free duals in order to explore the existence of free objects in the category of p-convex dual Banach lattices.

The talk is based on joint work with Pedro Tradacete [2].

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Fubini's theorem for a vector-valued Daniell integral

Jacobus J. Grobler
North-West University, Potchefstroom, South Africa
jacjgrobler@gmail.com

The vector-valued Daniell integral was defined in [1] following an idea of P.E. Protter [2]. Given two such integrals, we define their product Daniell integral and prove a Fubini theorem, for calculating the product integral as an iterated integral. In this we use the Fremlin vector lattice tensor product.

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Representation of quasi-unitary Banach lattices

Hamza Hafsi
University of Tunis, Tunisia
hafsi.hamza1@gmail.com

Truncated Riesz spaces, initially introduced by Fremlin in the realm of real-valued functions, were axiomatized by Ball. This paper focuses on the first axiom proposed by Ball, which serves as the definition of truncated Riesz spaces. The authors, in a previous work, demonstrated that if E is a truncated Riesz space, then $E \oplus \mathbb{R}$ can be endowed with a non-standard Riesz space structure. This construction ensures that E becomes a Riesz subspace of $E \oplus \mathbb{R}$, with truncation provided by the meet operation with 1.

In this study, we assume that the truncated Riesz space E possesses a lattice norm $\|\cdot\|$. We provide a necessary and sufficient condition for $E \oplus \mathbb{R}$ to have a lattice norm that extends $\|\cdot\|$. Furthermore, we establish that under this condition, the set of all lattice norms on $E \oplus \mathbb{R}$ that extend $\|\cdot\|$ essentially has a largest element denoted as $\|\cdot\|_1$ and a smallest element denoted as $\|\cdot\|_0$. Notably, any alternative lattice norm on $E \oplus \mathbb{R}$ is either equivalent to $\|\cdot\|_1$ or equal to $\|\cdot\|_0$.

As a consequence, we demonstrate that $E \oplus \mathbb{R}$ is a Banach lattice if and only if E is a Banach lattice. Additionally, our findings contribute to a representation theorem supported by the renowned Kakutani's Representation Theorem.

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Embedding of a truncated vector lattice into its universal completion

Rawaa Hajji

University of Tunis El Manar, Tunisia

rawaa.hajji@fst.utm.tn

We prove in a purely algebraic way that if L is an Archimedean truncated vector lattice then there exists a positive element e in the universal completion L^u of L such that the truncation of L is provided by meet with e. Previous representations of truncated vector lattices by almost-finite extended-real continuous valued functions can be obtained as consequences.

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A representation of variable exponent spaces $L^{p(\cdot)}(\mu)$ for infinite measures

Francisco L. Hernández

Madrid Complutense University, Spain

pacoh@ucm.es

It is presented a lattice isomodular representation of variable exponent (Nakano) spaces $L^{p(\cdot)}(\mu)$ with $p^+ < \infty$ for non-atomic separable σ-finite measures μ as a suitable variable exponent space $L^{q(\cdot)}$ on the $(0,1)$-interval and exponent functions having equal essential ranges. Some applications will be given.

The talk is based on joint work with C. Ruiz and M. Sanchiz.

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On Banach lattice algebras

Jamel Jaber
Carthage University, Tunisia
jamel.jaber@free.fr

In this talk we will discuss some problems on Banach lattice algebras. More precisely, we present some results concerning the positive projective tensor product, Arens regularity and existence of positive approximate unit.

An order theoretical analysis of atomic JBW algebras

Anke Kalauch
Dresden University of Technology, Germany
Anke.Kalauch@tu-dresden.de

Atomic JBW-algebras are known to be direct sums of JBW-algebra factors of type I. Extending Kadison's anti-lattice theorem, we establish that each of these factors is a disjointness free anti-lattice. We characterize order theoretical notions as disjointness, bands, and disjointness preserving bijections with disjointness preserving inverses in direct sums of disjointness free anti-lattices and, hence, in atomic JBW-algebras.

The talk is based on joint work with Mark Roelands and Onno van Gaans.

Orthogonality in ordered vector spaces

Anil Kumar Karn
National Institute of Science Education and Research Bhubaneswar, India
anilkarn@niser.ac.in

In this talk, we shall discuss a notion of orthogonality in ordered vector spaces. Every C*-algebra has a nice order structure. Though the self-adjoint part of a commutative C*-algebra is a vector lattice, its non-commutative counterpart is not, thanks to Kadison's anti-lattice theorem. We shall show that the proposed orthogonality unifies the two types of order structures. Further, we shall prove order theoretic characterizations of ℓ_p-spaces for $1 \leq p < \infty$ as well as that of c_0-spaces.

With the help of this orthogonality, we shall introduce a notion of absolute value in ordered vector spaces, namely, the notion of absolutely ordered spaces and that of absolute order unit spaces. Further, we shall describe orthogonality (absolute value) preserving maps and characterize unital surjective linear isometries between absolute order unit spaces. This extends Kadison's characterization of such isometries between unital C*-algebras and its counterpart to unital JB-algebras by Maitland Wright and Youngson. Such maps are precisely Jordan homomorphisms in the case of unital JB-algebras. Moreover, in a matricial version, these maps are exactly C*-algebra homomorphisms.

A part of the talk is based on joint work with Amit Kumar.

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The Choquet integral representability in a general setting

Jun Kawabe
Shinshu University, Japan
jkawabe@shinshu-u.ac.jp

Most of functionals, appeared in popular mathematical models for uncertainty and partial ignorance, are monotone, real-valued functionals defined on a vector sublattice of the space of $B(X)$ of all bounded, real-valued functions on a nonempty set X with additional properties such as the superadditivity, the comonotonic additivity, the translation invariance (or the constant additivity), and others. In those studies, it is important to clarify under what conditions a given functional I defined on a given vector sublattice Ψ of $B(X)$ can be represented as

$$I(f) = (\mathrm{C}) \int_X f d\mu, \quad f \in \Psi,$$

using the Choquet integral with respect to a nonadditive measure μ on X with appropriate regularity. This type of problem is often called the *Choquet integral representability* of a functional.

In this talk, using our improvement of the Greco theorem (1982) and König's scheme (1997), we give the Choquet integral representation theorem for continuous, comonotonically additive monotone functionals regardless of whether the space Ψ has the identity or not. Our representation theorem has the advantage that if the functional I is simultaneously continuous from above and below, then the representing nonadditive measure can also be constructed to be simultaneously continuous from above and below. Moreover, it covers the existing results for the same setting such as Giorgi and Letta (1977), Schmeidler (1986), Zhou (1998), Narukawa (2007), and Kawabe (2013).

Spectra of non-invertible weighted composition operators on uniform algebras

Arkady Kitover
Community College of Philadelphia, USA
akitover@ccp.edu

The main result presented in this talk is the following

Theorem. Let A be a unital uniform algebra. Let M_A and ∂_A be the space of maximal ideals and the Shilov boundary of A, respectively. Let φ be a continuous map of M_A into itself such that $\varphi(\partial_A) = \partial_A$, the restriction of φ on ∂_A is open, and such that the composition operator T_φ acts on A. Let $w \in A$, and let $T = wT_\varphi$. Assume that ∂_A has no isolated points.

Then $\sigma_{usf}(T, A) = \sigma_{a.p.}(T, A) = \sigma_{a.p.}(T, C(\partial_A))$, where $\sigma_{a.p.}(T, A)$ and $\sigma_{usf}(T, A)$ mean the approximate spectrum and the right semi-Fredholm spectrum of T, respectively

The talk is based on joint work with Mehmet Orhon.

Weak essential norms of pointwise multipliers between distinct Banach function spaces

Tomasz Kiwerski
Poznań University of Technology, Poland
tomasz.kiwerski@put.poznan.pl

Let us briefly recall that the quotient algebra $\mathcal{L}(X,Y)/\mathcal{W}(X,Y)$, that is, the ideal of bounded operators $\mathcal{L}(X,Y)$ modulo the ideal of weakly compact operators $\mathcal{W}(X,Y)$, is called the *weak Calkin algebra* and the corresponding quotient norm

$$\|T\colon X \to Y\|_w = dist(T\colon X \to Y, \mathcal{W}(X,Y)) \tag{1}$$

is called the *weak essential norm* of $T\colon X \to Y$. Moreover, by the space of *pointwise multipliers* $M(X,Y)$ between two Banach function spaces X and Y we understand a vector space of all functions, say f, with the property that the pointwise product fg belongs to Y for all $g \in X$. Since each function f from $M(X,Y)$ induces the multiplication operator $M_f\colon X \to Y$ given as $M_f\colon g \mapsto fg$, so it is natural to endow $M(X,Y)$ with the operator norm

$$\|f\|_{M(X,Y)} = \|M_f\colon X \to Y\| = \sup_{\|g\|_X=1} \|fg\|_Y. \tag{2}$$

Using quite recent Leśnik, Tomaszewski and Maligranda's result (see [3, Theorem 7]) characterizing Banach function spaces that satisfy the Dunford–Pettis criterion (that is, in which all relatively weakly compact sets are also uniformly integrable) as precisely those that possess the so-called positive Schur property (that is, in which every weakly null sequence with positive terms is norm convergent) along with some ideas from [1] we compute the weak essential norms of multiplication operators acting between two distinct Banach function spaces. More precisely, we will show (see [1, Proposition 2.6]) that for two Banach function spaces X and Y, the latter of which has the mentioned positive Schur property, we have the following formula

$$\|M_f\colon X \to Y\|_w = dist(f, M(X,Y)_o), \tag{3}$$

where $M(X,Y)_o$ stands for the separable part of $M(X,Y)$ (or, which is one thing, the ideal of all order continuous elements of $M(X,Y)$). Let us emphasize that the class of Banach function spaces with the positive Schur property includes, apart from such obvious things as L_1, some Lorentz spaces Λ_φ and Orlicz spaces L_M. If, in addition, both spaces X and Y are rearrangement invariant, then the above formula (3) describing the distance f from $M(X,Y)_o$ can be expressed in a fairly handy way, namely,

$$dist(f, M(X,Y)_o) = \lim_{n\to\infty} \|f^*\chi_{\left(0,\frac{1}{n}\right)\cup(n,\infty)}\|_{M(X,Y)}, \tag{4}$$

where f^* is the non-increasing rearrangement of f (see [1, Theorem 3.4] and [1, Corollary 2.17]).

The talk is based on collaborative work [1] with Jakub Tomaszewski from Poznań University of Technology.

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Quasi-modular spaces and copies of l^∞

Paweł Kolwicz
Poznań University of Technology, Poland
pawel.kolwicz@put.poznan.pl

We introduce the notion of a quasi-modular and we prove that the respective Minkowski functional of the unit quasi-modular ball becomes a quasi-norm. In this way, we refer to and complete the well-known theory related to the notion of a modular and a convex modular that lead to the F-norm and to the norm, respectively. Then we consider a special case of quasi-modular spaces, that is, the quasi-normed Calderón–Lozanovskiĭ spaces E_φ generated by a quasi-modular ρ_φ^E, where φ is a continuous, non-decreasing Orlicz function and E is a quasi-normed ideal space. Clearly, these spaces E_φ are generalizations of a quasi-normed Orlicz spaces. We present selected theorems concerning different copies (isomorphic or isometric) of l^∞ in the spaces E_φ in the natural language of suitable properties of the space E and the function φ. Denote by P a property of having of such a copy. We will show how the suitable conditions of the Orlicz function φ or the properties P of the quasi-Banach ideal space E affect the corresponding property P of the space E_φ. Moreover, we consider also the reverse implication. We will focus on those elements of this puzzle that distinguish the considered quasi-normed case from the well known normed case (which has been intensively and widely studied before).

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The talk is based on the joint work [1].

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Recent developments in nonlinear Perron-Frobenius theory

Brian Lins
Hampden-Sydney College, USA
blins@hsc.edu

A topical map is a function $T : \mathbf{R}^n \to \mathbf{R}^n$ that is order-preserving and additively homogeneous. Recent progress in nonlinear Perron-Frobenius theory has led to very general sufficient conditions to confirm the existence of additive eigenvectors for topical maps. We will present a computable necessary and sufficient condition for the set of additive eigenvectors to be nonempty and bounded with respect to the variation semi-norm. For topical maps with real analytic entries, these conditions also guarantee uniqueness of the eigevector (up to additive scaling). We will also describe conditions which imply that the (normalized) iterates $T^k(x)$ converge to an additive eigenvector of T for every $x \in \mathbf{R}^n$. Furthermore, when T is piecewise affine or when T is convex and real analytic, the rate of convergence is (at least) linear.

The positive cone of a Banach lattice. Coincidence of topologies and metrizability

Zbigniew Lipecki
Institute of Mathematics of the Polish Academy of Sciences, Poland
lipecki@impan.pl

Let X be a Banach lattice, and denote by X_+ its positive cone. The weak topology on X_+ is metrizable if and only if it coincides with the strong topology if and only if X is Banach-lattice isomorphic to $l^1(\Gamma)$ for a set Γ. The weak* topology on X_+^* is metrizable if and only if X is Banach-lattice isomorphic to a $C(K)$-space, where K is compact and metrizable.

Alexandroff unitization of a Lattice ordered algebra with a truncation

Mounir Mahfoudhi
University of Tunis El Manar, Tunisia
mounir.mahfoudhi@fst.utm.tn

Let R be a lattice ordered algebra along with a truncation in the sense of Ball. We give a necessary and sufficient condition on R for its unitization R^* in order to be again a lattice ordered algebra. Also, we shall see that R^* is a lattice ordered algebra for at most one truncation. A special attention is paid to the Archimedean case. More precisely, we shall identify the unique truncation on an Archimedean ℓ-algebra R which makes R^* into a lattice ordered algebra.

The talk is based on joint work with Karim Boulabiar.

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Riesz Space Generalizations of recurrence Theorems in Ergodic Theory

Marwa Masmoudi
University of Tunis El Manar, Tunisia
marwa_masmoudi@hotmail.com

We will develop Riesz space versions of the Poincaré Recurrence Theorem and the Kac formula which are fundamental results dealing with recurrence in ergodic theory. Additionally, we will give a formulation of the Kakutani-Rokhlin decomposition in terms of components of weak order units of Riesz spaces. As an application, we will prove that every aperiodic transformation can be approximated by a periodic one.

The talk is based on joint work with Y. Azouzi, M.A. Ben Amor, J.M. Homann and B.A. Watson.

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Grothendieck compactness principle for the absolute weak topology

Vinícius Miranda
Universidade Federal de Uberlândia, Brasil
colferaiv@gmail.com

The compactness principle proved by Grothendieck states that every norm compact subset of a Banach space is contained in the closed convex hull of a norm null sequence. The authors in [2] studied a version of this principle concerning the weak topology and proved the following outstanding result: every weakly compact subset of a Banach space is contained in the closed convex hull of a weakly null sequence if and only if the Banach space has the Schur property. As expected, Grothendieck compactness-type principles have been considered for different topologies (see the references in [1]). This talk corresponds to the results obtained in the preprint [1] where we proved the Grothendieck's compactness principle for the absolute weak topology in Banach lattices. In our way to prove this result, we realized that some well known results on the weak topology on Banach spaces that are often used in the proofs of compactness principles have no known analogues for the absolute weak topology on Banach lattices. The result is that we were forced to prove lattice versions for the absolute weak topology of known results for the weak topology:
(1) Every absolutely weakly compact set in a Banach lattice is absolutely weakly sequentially compact.
(2) The converse of (1) holds if E is separable or $B_{E^{**}}$ is absolutely weak* compact.

With these two results we proved our main result:
(3) A Banach lattice E has the positive Schur property if and only if every absolutely weakly compact subset of E is contained in the closed convex hull of an absolutely weakly null sequence.

As an application of (3) we give a characterization of the dual positive shur property concerning sequentially absolutely weak* compact sets.

The talk is based on joint work with Geraldo Botelho and José Lucas P. Luiz.

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Cones with semi-interior points and Applications

Ioannis A. Polyrakis
National Technical University of Athens, Greece
ypoly@math.ntua.gr

Suppose that P is a cone of a normed space X. A vector $x_0 \in P$ is a semi-interior point of P if a real number $\rho > 0$ exists so that $x_0 - \rho U_+ \subseteq P$, where $U_+ = P \cap U$ is the positive part of the unit ball U of X. In this article we give examples and we study properties of cones with semi-interior points in normed spaces and in topological vector spaces. Also we give a applications of semi-interior points in Economics (General Equilibrium) and in E-metric spaces. Note that the notion of semi-interior point has been defined by the speaker and has been announced in his talk during the XXII European Workshop on General Equilibrium Theory, in Paris, 2014. This definition with some results of this theory have been published in [3].

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Generalized ergodic domination in ordered Banach algebras

Dimby Rabearivony
Stellenbosch University, South Africa
dimbir@aims.ac.za

In ordered Banach algebra (OBA) theory, various authors have studied the so-called domination problem (see, e.g., Section 4.2 in [2]): given two elements a and b of an OBA such that $0 \leq a \leq b$, under what hypotheses are properties of b inherited by a? We tackle the corresponding problem where the condition $0 \leq a \leq b$ is replaced by the weaker condition $\pm a \leq b$. (For operators S and T on a Banach lattice E the condition $\pm S \leq T$ means that $|Sz| \leq T|z|$ for all z in E.) We refer to this as the generalized domination problem. Furthermore, it is presented as an open question in [2] whether the (known) ergodic domination theorem (see [1]) can be extended to this setting. We will show that this question has a positive answer, not only for ergodic domination, but also for most of the existing domination results, including those related to Riesz elements, inessential elements and elements of the radical.

The talk is based on joint work with Sonja Mouton (Mathematics Division, Stellenbosch University, South Africa).

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Totally bounded sets in locally convex cones

Asghar Ranjbari
University of Tabriz, Iran
ranjbari@tabrizu.ac.ir

A locally convex cone is an structure which its topology is based on an order (see [1] or [3]). There are three types of topologies in a locally convex cone i.e. lower, upper and symmetric topologies. A totally bounded set with respect to these topologies were defined in [1] and [2]. We verify some properties of these sets in locally convex cones. Also, we consider locally convex semilattice cones and investigate totally boundedness of supremum and infimum of two sets in this cones.

The talk is based on joint work with Saeed Vazifeh.

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Functions operating on several multivariate distribution functions

Paul Ressel
Katholische Universität Eichstätt-Ingolstadt, Germany
paul.ressel@ku.de

Functions f on $[0,1]^m$ such that every composition $f \circ (g_1, \dots, g_m)$ with d-dimensional distribution functions $g_1, \dots, g_m$ is again a distribution function, turn out to be characterized by a very natural monotonicity condition, which for $d = 2$ means ultramodularity. For $m = 1$ (and $d = 2$) this is equivalent with increasing convexity.

Prime ideals and Noetherian properties in vector lattices

Mark Roelands
Leiden University, Netherlands
m.roelands@math.leidenuniv.nl

In this talk, we consider the set of all prime ideals in vector lattices and how their properties structure the vector lattice. Firstly, we discuss the situation where there are only finitley many prime ideals. Secondly, when every prime ideal is principal, which yields a lattice version of a result by Cohen and Kaplansky for commutative rings. Lastly, when every ascending chain of prime ideals is stationary, the prime Noetherian property.

The talk is based on joint work with Marko Kandić.

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Weak compactness in Variable Lebesgue spaces

Mauro Sanchiz
Universidad Complutense de Madrid, Spain
msanchiz@ucm.es

Variable Lebesgue spaces (or Nakano spaces) $L^{p(\cdot)}(\Omega)$ are a generalization of classical Lebesgue spaces, and a particular class of non-symmetric Musielak-Orlicz spaces. They have seen a strong renewed relevance in the last decades, due to their applications to harmonic analysis and differential equations.

In this talk, we show criteria for a subset of $L^{p(\cdot)}(\Omega)$ be equi-integrable or relatively weakly compact for bounded exponents $1 \leq p^- \leq p^+ < \infty$, using as references classical results for Lebesgue spaces L_p (de la Vallée-Poussin, Dunford-Pettis) and Orlicz spaces L^φ (Luxemburg, Andô). From them, we get consequences as a characterization for sequences (f_n) in $L^{p(\cdot)}(\Omega)$ to be weakly convergent to $f \in L^{p(\cdot)}(\Omega)$ and that $L^{p(\cdot)}(\Omega)$ is weakly Banach-Saks if and only if $p^+ < \infty$.

In particular, an (Andô's type) criterion for a subset $S \subset L^{p(\cdot)}(\Omega)$ (for either a finite or σ-finite separable measure space Ω) is relatively weakly compact if and only if

$$\lim_{\lambda \to 0} \sup_{f \in S} \frac{1}{\lambda} \int_\Omega |\lambda f(t)|^{p(t)} d\mu = 0.$$

The talk is based on joint work with Francisco L. Hernández and César Ruiz.

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The essential spectrum, norm, and spectral radius of abstract multiplication operators

Anton R. Schep
University of South Carolina, USA
schep@math.sc.edu

Let E be a complex Banach lattice and T is an operator in the center $Z(E) = \{T : |T| \leq \lambda I \text{ for some } \lambda\}$ of E. Then the essential norm $\|T\|_e$ of T equals the essential spectral radius $r_e(T)$ of T. We also prove $r_e(T) = \max\{\|T_{A^d}\|, r_e(T_A)\}$, where T_A is the atomic part of T and T_{A^d} is the non-atomic part of T. Moreover $r_e(T_A) = \limsup_{\mathcal{F}} \lambda_a$, where $\mathcal{F}$ is the Fréchet filter on the set A of all positive atoms in E of norm one and λ_a is given by $T_A a = \lambda_a a$ for all $a \in A$. Some partial results for disjointness preserving maps will be mentioned.

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Irreducible local representations and pure local completely positive and local completely contractive maps of locally C^*-algebras

Ionuţ Şimon
Politehnica University of Bucharest, Romania
ionutsimon.gh@gmail.com

The notion of irreducible local representation of a locally C^*-algebra is introduced and an appropriate analogue of purity of local completely positive maps on locally C^*-algebras is obtained.

A localization principle in pre-Riesz spaces

Janko Stennder
Dresden University of Technology, Germany
janko.stennder@tu-dresden.de

In Archimedean vector lattices, it is well known that every principle ideal can be represented as a uniformly dense sublattice of some $\mathrm{C}(K)$ space. Therefore, by restricting to a suitable principle ideal, calculations can be done concretely using continuous functions. We introduce and investigate a similar localization principle in pre-Riesz spaces, involving the functional representation of order unit spaces.

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Nuclear operators and operator-valued Borel measures

Juliusz Stochmal
Kazimierz Wielki University in Bydgoszcz, Poland
j.stochmal@ukw.edu.pl

The concept of a nuclear operator between Banach spaces is due to Ruston and Grothendieck and has the origin in the Schwartz's kernel theorem.

Let E and F denote Banach spaces. For the Banach space $C(K, E)$ of continuous E-valued functions on a compact Hausdorff space K, the study of nuclear operators $T : C(K, E) \to F$ was initiated by Alexander, where the result of Schwartz was extended in case E' has the Radon–Nikodym property. In [3] Saab and Smith showed that the condition on E' to have the Radon–Nikodym property is necessary. Moreover, the study of nuclear operators on $C(K, E)$ has been developed by Popa [2].

Let X be a completely regular Hausdorff space and let $C_b(X, E)$ stand for the space of E-valued bounded continuous functions on X, equipped with the strict topology β.

The aim of this talk is to characterize nuclear operators $T : C_b(X, E) \to F$ between the locally convex space $(C_b(X, E), \beta)$ and the Banach space F in terms of their representing operator-valued Borel measures. We also study the relationship between the nuclearity of T and the nuclearity of its conjugate operator T'.

The talk is based on joint work with Nowak [1] and [4].

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Essential norms of pointwise multipliers between distinct Köthe spaces

Jakub Tomaszewski
Poznań University of Technology, Poland
jakub.tomaszewski@put.poznan.pl

Motivated by some recent results, but also referring to classics, we compute the essential norm[1] of the multiplication operators[2] acting between two distinct Köthe spaces. More precisely, for two Banach sequence spaces X and Y such that either the space Y is separable or the space X is reflexive, the essential norm of the multiplication operator $M_\lambda\colon X \to Y$ is given by

$$\|M_\lambda\colon X \to Y\|_{ess} = \lim_{n\to\infty} \|\lambda\chi_{\{n,n+1,\dots\}}\|_{M(X,Y)}.$$

Moreover, we will show that for two Banach function spaces X and Y such that either the space X is separable or the space Y is reflexive the essential norm is always maximal, that is,

$$\|M_\lambda\colon X \to Y\|_{ess} = \|M_\lambda\colon X \to Y\|_{M(X,Y)}.$$

The talk is based on joint work [1] with Tomasz Kiwerski from Poznań University of Technology.

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[1]Recall that the **essential norm** of an operator $T\colon X \to Y$ is given by the formula

$$\|T\colon X \to Y\|_{ess} = \inf\{\|T-K\|_{X\to Y} : K \in \mathcal{K}(X,Y)\},$$

where $\mathcal{K}(X,Y)$ stands for the ideal of compact operators.

[2]For a given pair of Köthe spaces X and Y defined over the same measure space and a function λ by the **multiplication operator** $M_\lambda\colon X \to Y$ we understand the mapping

$$M_\lambda x = \lambda x \quad \text{for } x \in X.$$

The space of all functions λ for which $M_\lambda\colon X \to Y$ is bounded, equipped with the operator norm, is denoted by $M(X,Y)$.

On ℓ-algebras satisfying $a \wedge b = 0 \Rightarrow (ab)^2 = 0$

Ayşe Uyar
Gazi University, Turkey
ayseu@gazi.edu.tr

We introduce a "new" class of lattice ordered algebras, which will be called an $\mathfrak{a}$-algebra. We present some properties of $\mathfrak{a}$-algebras and consider their relationships with various type of lattice ordered algebras. We improve a result of Huijsmans. It is shown that if A is uniformly complete $\mathfrak{a}$-algebra with unit $e > 0$ then ${B_e}^d$ is (two- sided) ℓ-ideal and also ${B_e}^d = N(A) = \{a \in A : a^2 = 0\}$. It is obtained that if T is an operator from an uniformly complete $\mathfrak{a}$-algebra A with positive unit e_A to an f-algebra B with unit e_B then T is an algebra homomorphism iff T is Riesz homomorphism. Also, it is shown that the order continuous order bidual $(A')'_n$ of an Archimedean $\mathfrak{a}$-algebra A is an $\mathfrak{a}$-algebra with respect to Arens multiplication.

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Free objects in analytic categories

Walt van Amstel
University of Pretoria, South Africa
sjvdwvanamstel@gmail.com

In recent years, the notion of a *free Banach lattice* over various different structures has become a very active area of research. Their existence and properties have been studied in [1], [2], [4], [5], and [6], amongst many others. The existence of these free objects is most often established by means of a concrete model. In [3], relevant material from the theory of universal algebra is presented to establish the existence of free vector lattices and free vector lattice algebras.

We will discuss a uniform approach to the construction of free objects in categories of normed structures. This will make use of the abstract existence of free objects in algebraic categories established in [3]. In particular, this approach will establish the existence of ((positive) unital) Banach lattice algebras, the existence of which (to the presenter's knowledge) is yet to be established by means of a concrete model.

Strictly speaking, these free objects in categories of normed structures are not 'free' in the sense of category theory since only 'bounded' maps can be factored through the free object.

We will see, however, that these 'free' objects can be combined by means of an inverse limit to obtain genuine free objects in categories of locally convex structures.

This is joint work with Marcel de Jeu from Leiden University.

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Hilbert geometries isometric to Banach spaces

Cormac Walsh

CMAP Ecole Polytechnique, France

cormac.walsh@inria.fr

The Hilbert metric can be defined on the interior of the cone of any order-unit space, and encodes important geometric information about the cone. It was observed by Nussbaum and de la Harpe that the Hilbert geometry on the standard positive cone $\mathbb{R}^n_+$ is actually isometric to a normed space, and later it was proved by Foertsch and Karlsson that these are the only finite-dimensional cones with this property.

In this talk, I will show how to extend this result to infinite dimension, that is, prove that a Hilbert geometry is isometric to a Banach space if and only if the cone it is defined on is the cone of positive continuous functions on some compact Hausdorff space.

The main technique in the proof is to compare the horofunction boundaries of Hilbert geometries and of Banach spaces. I will recall what this boundary is and what it looks like for these two types of space.

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Representing Noetherian vector lattices

Marten Wortel

University of Pretoria, South Africa

marten.wortel@up.ac.za

A vector lattice is Noetherian if every increasing sequence of ideals stabilizes. We will discuss a representation theorem for Noetherian vector lattices. The main tools used are lexicographic vector lattices, local and semi-local ideals, and well-founded recursion.

The talk is based on joint work with Marko Kandić and Mark Roelands.