Summary of exercises / Additional exercises

Topic 1

Exercise: Consider the usual drawing of the prism $K_2 \square C_n$, $n \geq 3$, (cartesian product) in the plane.

(a) If we make every K_2 edge twisted, what is the Euler genus of the corresponding embedding (use Euler's formula!), and is it orientable or nonorientable?

(b) Suppose we instead make every edge of one copy of C_n twisted, but leave all other edges untwisted. Answer the same question.

Topic 3

Exercise: Consider the current graph from Ringel's Fig. 9.1. Trace the faces and determine the derived graph. Also determine the distribution of face degrees in the derived embedding.

In his book Ringel uses this current graph to construct a triangular embedding of $K_{14} - K_2$, the graph obtained by deleting one edge from K_{14}. How does this work? [Hints: keep only one component; add vertices in large faces.]

Exercise: See notes for problem on voltages on gems.

Topic 4

Exercise: See notes for problem on nonorientable genus of $K_{m,n}$.

Exercise: Suppose we can construct an orientable hamilton cycle embedding of $K_{n,n,n}$ for some particular n. For what family of graphs (as large as possible) can we then use the diamond sum to obtain minimum orientable genus embeddings?

Repeat the question for $K_{n,n,n,n}$.

Topic 5

Exercise from notes, expanded and corrected: Find a transition graph that generates a *nonorientable* embedding of $K_{14,14}$ with twelve hamilton cycle faces and all other faces being 4-cycles. [Note: using any V pattern guarantees that you have a nonorientable embedding.]

Now repeat for eleven hamilton cycle faces.

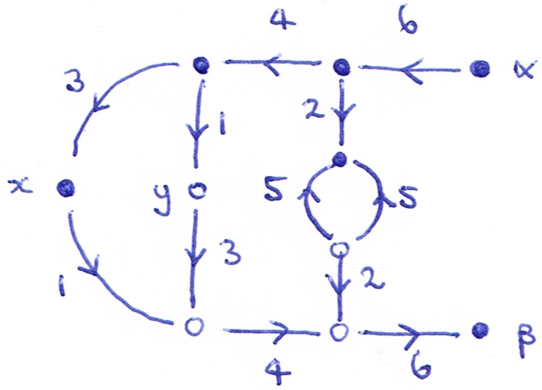
These allow us to get minimum nonorientable genus embeddings of $K_{14,14,12}$ and $K_{14,14,11}$, by adding vertices in the hamilton cycle faces. Can you set up your embeddings so that by using 2H $\leftrightarrow$ V transformations you can also get minimum genus nonorientable embeddings of $K_{14,14,t}$ for some other values of t? Set up your original embeddings so that you can cover as many t as possible in this way.

Now (if you are not worn out) repeat for *orientable* embeddings. Besides having different original embeddings, you should use 4H $\leftrightarrow$ 2X transformations instead of 2H $\leftrightarrow$ V transformations.

Topic 6

Exercise: Use adding a crosscap around a vertex to transform the embedding of $K_{4,4}$ on the torus given on page 13 of the notes into an embedding of $K_{4,5}$ on N_3.

Ringel's Fig. 9.1 (slightly modified)



$\mathbb{Z}_{12}$